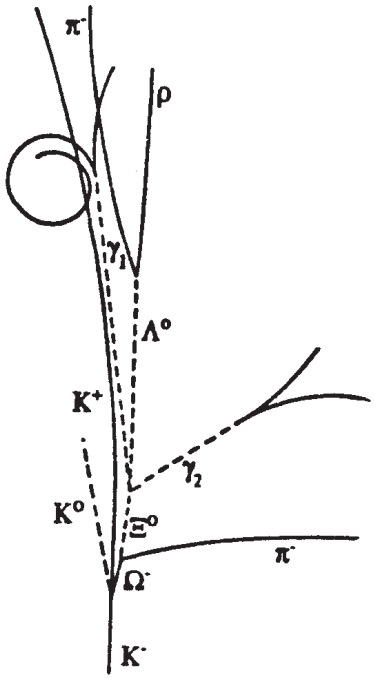
VOLUME 48, NUMBER 2, JUNE 2025

HADRONIC JOURNAL Volume 48, Number 2, June 2025

HADRONIC JOURNAL

Founded in 1978 by Prof. R. M. Santilli at Harvard University. Some of the past Editors include Professors S. L. Adler, A. 0. Barut, L. C. Biedenharn, N. N. Bogoliubov, M. Froissart, J.Lohmus, S. Okubo, Nobel Laureate Ilya Prigogine, M. L. Tomber, J. P.Vigier, J. Wess, Nobel Laureate Chen Ning Yang.



EDITORIAL BOARD

A.O.ANIMALU
A. K. ARINGAZIN
A. A. BHALEKAR
S. J. DHOBLE
J. DUNNING-DAVIES
T. L. GILL
L. P. HORWITZ
S. L. KALLA
S. I. KRUGLOV
J. LISSNER
M. NISHIOKA
R. F. O'CONNELL
Z. OZIEWICZ*
E. RECAMI*
M. SALEEM
S. SILVESTROV
H. M. SRIVASTAVA
E. TRELL
R.I. TSONCHEV
QI-REN ZHANG
C.A.WOLF
YI-ZHONG ZHUO

*In memoriam

FOUNDER and Editor In Chief
R. M. SANTILLI

HADRONIC PRESS, INC.

HADRONIC JOURNAL

Established in 1978 by Prof. R. M. Santilli at Harvard University Hadronic Journal and Algebras, Groups and Geometries have been regularly published since 1978 without publication charges and are among the few remaining independent refereed journals.

This Journal publishes advances research papers and Ph. D. theses in any field of mathematics without publication charges.

For subscription, format and any other information please visit the website
http://www.hadronicpress.com

HADRONIC PRESS INC.
35246 U. S. 19 North Suite 215
Palm Harbor, FL 34684, U.S.A.
http://www.hadronicpress.com
Email: info@hadronicpress.com
Phone: +1-727-946-0427

HADRONIC JOURNAL

Founder and Editor in Chief
RUGGERO MARIA SANTILLI
The Institute for Basic Research
35246 U. S. 19 North Suite 215, Palm Harbor, FL 34684, U.S.A.
Email: resarch@i-b-r.org; TEL: +1-727-688-3992

A. O. ANIMALU, University of Nigeria, Department of Physics, Nsukka, Nigeria animalu@nmc.edu.ng
A.K. ARINGAZIN, Department of Theoretical Physics, Institute for Basic Research Eurasian National University Astana 010008, Kazakhstan aringazin@mail.kz
A.A. BHALEKAR, Department of Chemistry, R.T.M. Nagpur University, Nagpur, 440033 India anabha@hotmail.com
S.J. DHOBLE, Department of Physics R.T.M. Nagpur University Nagpur, 440033 India sjdhoble@rediffmail.com
J. DUNNING-DAVIES, Department of Physics (Retired) University of Hull Hull, HU6 7RX England j.dunning-davies@hull.ac.uk
T. L. GILL, Howard University Research Center ComSERC Washington, DC 20059, USA tgill@howard.edu
L. P. HORWITZ, Department of Physics Tel Aviv Univ., Ramat Aviv, Israel horwitz@taunivm.tau.ac.il
S.L. KALLA, Department of Mathematics Vyas Institute of Higher Education Jodhpur, 342008, India shyamkalla@gmail.com
S.I. KRUGLOV, University of Toronto at Scarborough, Physical and Environmental Sciences Dept., 1265 Military Trail, Toronto, Ontario, Canada M1C 1A4 skrouglo@utm.utoronto.ca
J. LISSNER, Alumnus Foukzon Laboratory Center for Mathematical Sciences Technion-Israel Institute of Technology Haifa, 3200003, Israel
M. NISHIOKA, Yamaguchi University Department of Physics, Yamaguchi 753, Japan
R. F. O'CONNELL, Louisiana State University Department of Physics, Baton Rouge, LA 70803
Z.OZIEWICZ,* Universidad National Autonoma de Mexico, Faculdad de Estudios Superiores C.P. 54714, Cuautitlan Izcalli Aparto Postal # 25, Mexico **** In memoriam***
E. RECAMI, Universita' de Bergamo, Facolta' di Ingeneria, Viale Marconi 5, 1-24044 Dalmine (BG) Italy **** In memoriam***
M. SALEEM, University of the Punjab Center for High Energy Phys., Lahore, Pakistan dms@lhr.paknet.com.pk
S. SILVESTROV, School of Education, Culture and Communication (UKK) Malardalen University Box 883, 71610 Västerås, Sweden sergei.silvestrov@mdh.se
H. M. SRIVASTAVA, Department of Mathematics and Statistics, University of Victoria, Victoria, B. C. V8W 3P4, Canada, hmsri@uvvm.uvic.ca
E. TRELL, Faculty of Health Sciences, University of Linkoping, Se-581 83, Linkoping, Sweden erik.trell@gmail.com
R.I. TSONCHEV, Facultad de Fisica, Universidad Autonoma de Zacatecas, P. O. C-580, Zacatecas 98068, zac, Mexico rumen@ahobon.reduaz.ms
QI-REN ZHANG, Peking University, Department of Technical Phys., Beijing 100871, China zhangqr@sun.ihep.ac.cn
C.A. WOLF, Department of Physics, Massachusetts College of Liberal Arts, North Adams, Ma 01247 cwolf@mcla.mass.edu
YI-ZHONG ZHUO, Institute of Atomic Energy P.O. Box 275 (18) Beijing 102413, China zhuoyz@mipsa.ciae.ac.cn

HADRONIC JOURNAL

VOLUME 48, NUMBER 2, JUNE 2025

VARIATIONAL FORMULATIONS FOR NON-RELATIVISTIC AND RELATIVISTIC FLUID MOTIONS IN AN EULERIAN CONTEXT, 125

Fabio Silva Botelho
Department of Mathematics
Federal University of Santa Catarina – UFSC
Florianópolis, SC, Brazil

TOWARDS AN EQUATION OF EVERYTHING IN THE THREE-DIMENSIONAL DYNAMIC QUANTUM VACUUM, 147

Davide Fiscaletti
SpaceLife Institute – Via Roncaglia 35
S. Lorenzo in Campo (PU), Italy

MASSES OF MODIFIED ATOMS OF TUNGSTEN ISOTOPES, 185

Mikhail Kashchenko
Ural Federal University, Yekaterinburg, Russia
Ural State Forestry University, Yekaterinburg, Russia
Maksim Kovalenko
Ural Federal University, Yekaterinburg, Russia
Vladimir Pechorsry
Ural State Forestry University, Yekaterinburg, Russia
Nadezhda Kashchenko
Ural Federal University, Yekaterinburg, Russia

THERMODYNAMICS IN n-SPATIAL DIMENSIONS, 199

J. J. Bevelacqua
Bevelacqua Resources
7531 Flint Crossing Circle SE
Owens Cross Roads, AL 35763

UNDERSTANDING THE ROLE OF NUCLEAR ELEMENTARY CHARGE AND ITS POTENTIAL IN ESTIMATING NUCLEAR BINDING ENERGY AND STRONG COUPLING CONSTANT, 207

U. V. S. Seshavatharam
Honorary Faculty, I-SERVE, Survey no-42, Hitech City
Hyderabad-84, Telangana, India
S. Lakshminarayana
Department of Nuclear Physics, Andhra University
Visakhapatnam-03, AP, India

THE ROLE OF CONSCIOUSNESS FOR GRAND UNIFIED THEORIES, 231

Bertrand Wong
Dept. of Science & Technology
Eurotech, Singapore Branch

HADRONIC JOURNAL 48, 125 - 145 (2025)
DOI 10.29083/HJ.48.02.2025/SC125

VARIATIONAL FORMULATIONS FOR NON-RELATIVISTIC AND RELATIVISTIC FLUID MOTIONS IN AN EULERIAN CONTEXT

Fabio Silva Botelho
Department of Mathematics
Federal University of Santa Catarina – UFSC
Florianópolis, SC, Brazil
fabio.botelho@usfc.br

Received October 3, 2024

Abstract

In its first part, this article develops a variational formulation for a fluid motion in a non-relativistic context. The results are obtained through standard tools of calculus of variations and constrained optimization in function spaces. In order to include a scalar field of temperature as a variable, concerning the main functional, we assume a rather standard energy equation as a constraint. In the last section, we also address a fluid motion in a relativistic context. Finally, we emphasize the modeling approach here addressed is essentially an Eulerian one.

AMS Classification 76A02, 76N15
Keywords. Variational formulation, compressible Euler system, temperature scalar field.

Copyright © 2025 by Hadronic Press, Inc., Palm Harbor, FL 34684, USA

1 Introduction

In its first part, this article develops a variational formulation for a fluid motion in a non-relativistic context.

We assume a total mass restriction as a constraint along a period of time denoted by $[0, t_f]$.

The results are obtained through standard tools of calculus of variations, basic constrained optimization and related Lagrange multipliers approach in infinite dimensional spaces.

As standard references in theoretical fluid mechanics we would cite [1, 2, 3, 4, 5, 6]. For related numerical methods we would mention [7, 8, 9]. Concerning similar models, we would cite [10, 11, 12, 13, 14].

Finally, details on the Sobolev spaces involved may be found in [15].

Let $\Omega \subset \mathbb{R}^3$ be an open, bounded and connected set with a regular (Lipschitzian) boundary denoted by $\partial\Omega$.

Consider a fluid motion initially modeled as a particle system one, through the field of position $\mathbf{r} : \Omega \times [0, t_f] \to \mathbb{R}$ and field of velocities $\mathbf{v}$, where $\mathbf{v}(\mathbf{r}(x,t),t)$ stands for the velocity at the position $\mathbf{r}(x,t)$ and time $t \in [0, t_f]$.

2 A variational formulation for a compressible fluid motion

In this section, we develop in details the concerning variational formulation for a compressible case.

Assuming the Einstein convection of summing up repeated indices and denoting

$$\mathbf{r}(x,t) = (X_1(x,t), X_2(x,t), X_3(x,t)),$$

for such a compressible motion we also assume the mass conservation equation as a constraint, which stands for

$$\frac{d\rho}{dt} + \rho \frac{\partial}{\partial x_j}\left(\frac{\partial X_j(x,t)}{\partial t}\right) = 0, \text{ in } \Omega \times [0, t_f].$$

Moreover, we suppose a non-constant field of temperature $T(\mathbf{r}(x,t),t)$ and a non-constant fluid density $\rho = \rho(\mathbf{r}(x,t),t) > 0$ so that the macroscopic system kinetics energy functional stands for

$$E_c = \frac{1}{2}\int_0^T \int_\Omega \rho \mathbf{v}(\mathbf{r}(x,t),t) \cdot \mathbf{v}(\mathbf{r}(x,t),t)\, dx\, dt.$$

For such a modeling, denoting by $\mathbf{g} = -g\mathbf{k}$ the gravity field, we assume such a system is a particle one, comprised by a number N of particles, with position fields

$$\mathbf{r}_j(x,t) = \mathbf{r}(x,t) + \hat{\mathbf{r}}_j(\mathbf{r}(x,t),t),\ \forall j \in \{1,\cdots,N\}.$$

Here $\hat{\mathbf{r}}_j(\mathbf{r}(x,t),t)$ refers to internal vibrational degrees of freedom related to the internal energy and temperature concepts.

We assume the following constraints,

$$C_v T = \sum_{j=1}^{N} \frac{1}{2}|\phi_j|^2 \frac{d\hat{\mathbf{r}}_j(\mathbf{r}(x,t),t)}{dt} \cdot \frac{d\hat{\mathbf{r}}_j(\mathbf{r}(x,t),t)}{dt},$$

in $\Omega \times [0,t_f]$, where

$$|\phi_j(\mathbf{r}(x,t),t)|^2$$

is the scalar density field of the particle p_j, and following the approach in reference [16],

$$C_v\rho\frac{dT(\mathbf{r}(x,t),t)}{dt} - K\nabla^2 T - \rho\mathbf{g}\cdot\mathbf{v} + P\ \text{div}\ (\mathbf{v}) - \frac{\partial Q}{\partial t} = 0,$$

in $\Omega \times [0,t_f]$, where as previously indicated,

$$T = T(\mathbf{r}(x,t),t)$$

is the temperature scalar field, $C_v > 0$, $K > 0$ are real constants and Q is a heat function.

We define also the mass restrictions,

$$\int_\Omega |\phi_j(\mathbf{r}(x,t),t)|^2\, dx = m_{p_j},\ \text{in}\ [0,t_f],$$

$$\rho(\mathbf{r}(x,t),t) = \sum_{j=1}^{N} |\phi_j(\mathbf{r}(x,t),t)|^2,\ \text{in}\ \Omega \times [0,t_f],$$

where m_{p_j} is the mass of a particle p_j.

As usual, we generically denote

$$\langle f,h\rangle_{L^2} = \int_0^{t_f}\int_\Omega f\ h\ dxdt,\ \forall f,h \in L^2(\Omega \times [0,t_f]),$$

with an analogous notation for a vectorial case.

Concerning such a fluid motion modeling, denoting generically $Y = C^1$, $Y_1 = C^2$ and $V = C_0^2$, we define the following variational formulation represented by

the functional $J : Y \times V \times Y_1 \to \mathbb{R}$ already including the concerning Lagrange multipliers for the constraints, where

$$
\begin{aligned}
& J(\mathbf{v}, \mathbf{r}, \hat{\mathbf{r}}, \phi, \rho, T, \lambda, \hat{P}, E) \\
= \; & \frac{1}{2} \int_0^{t_f} \int_\Omega \rho \mathbf{v}(\mathbf{r}(x,t),t) \cdot \mathbf{v}(\mathbf{r}(x,t),t) \; dx \; dt \\
& + \frac{1}{2} \sum_{j=1}^{N} \int_0^{t_f} \int_\Omega |\phi_j(\mathbf{r}(x,t),t)|^2 \frac{d\hat{\mathbf{r}}_j(\mathbf{r}(x,t),t)}{dt} \cdot \frac{d\hat{\mathbf{r}}_j(\mathbf{r}(x,t),t)}{dt} \; dxdt \\
& + \sum_{j=1}^{N} \int_0^{t_f} \int_\Omega |\phi_j(\mathbf{r}(x,t),t)|^2 \frac{d\hat{\mathbf{r}}_j(\mathbf{r}(x,t),t)}{dt} \cdot \frac{d\mathbf{r}(x,t)}{dt} \; dxdt \\
& + \left\langle \lambda, \mathbf{v}(\mathbf{r}(x,t),t) - \frac{\partial \mathbf{r}(x,t)}{\partial t} \right\rangle_{L^2} \\
& - \left\langle \hat{P}, \frac{d\rho}{dt} + \rho \frac{\partial}{\partial x_j} \left(\frac{\partial X_j(x,t)}{\partial t} \right) \right\rangle_{L^2} \\
& - \int_0^{t_f} \int_\Omega \rho \mathbf{g} \cdot \mathbf{r}(x,t) \; dxdt \\
& + \int_0^{t_f} \int_\Omega \lambda_1 \left(\rho \, C_v \, \frac{d\,T}{dt} - K \nabla^2 T + P \; \mathrm{div} \; \mathbf{v} - \rho \mathbf{g} \cdot \mathbf{v} \right) \; dxdt \\
& + \int_0^{t_f} \int_\Omega \lambda_2 \left(C_v \, T - \frac{1}{2} \sum_{j=1}^{N} |\phi_j|^2 \frac{d\hat{\mathbf{r}}_j(\mathbf{r}(x,t),t)}{dt} \cdot \frac{d\hat{\mathbf{r}}_j(\mathbf{r}(x,t),t)}{dt} \right) \; dxdt \\
& + \frac{1}{2} \sum_{j=1}^{N} \gamma_j \int_0^{t_f} \int_\Omega \nabla \phi_j \cdot \nabla \phi_j \; dxdt \\
& + \frac{1}{2} \sum_{j=1}^{N} \gamma_0 \int_0^{t_f} \int_\Omega \frac{d\phi_j}{dt} \frac{d\phi_j}{dt} \; dxdt \\
& + \sum_{j=1}^{N} \sum_{k=1}^{N} \int_0^{t_f} \int_\Omega \int_\Omega \alpha_{jk} \frac{|\phi_j(\mathbf{r}(x,t),t)|^2 |\phi_j(\mathbf{r}(\tilde{x},t),t)|^2}{|\mathbf{r}(x,t) - \mathbf{r}(\tilde{x},t)|} \; dx \; d\tilde{x} \; dt \\
& - \sum_{j=1}^{N} \int_0^{t_f} E_j(t) \left(\int_\Omega |\phi_j(\mathbf{r}(x,t),t)|^2 \; dx - m_{p_j} \right) \; dt \\
& + \int_0^{t_f} \int_\Omega \lambda_3 \left(\rho(\mathbf{r}(x,t),t) - \sum_{j=1}^{N} |\phi_j(\mathbf{r}(x,t),t)|^2 \right) \; dxdt. \qquad (1)
\end{aligned}
$$

where the pressure field P is defined by

$$P = P(\hat{P}, \rho) = \frac{d(\hat{P}\rho)}{dt},$$

$$\gamma_j > 0, \ \forall j \in \{1, \cdots, N\}$$

and $\gamma_0 > 0$ are appropriate real constants, and

$$\{\alpha_{jk}\}_{N \times N}$$

is an appropriate positive definite real matrix.

3 The passage from a discrete model to a continuous one

Observe that in general N is very large.

With such an issue in mind, we shall propose to replace the discrete variables

$$\{\hat{\mathbf{r}}_j(\mathbf{r}(x,t),t), \ j \in \{1, \cdots, N\}\},$$

by a continuous one in s

$$\hat{\mathbf{r}}(\mathbf{r}(x,t),t,s)$$

where $s \in [0,b]$, for some appropriate large real constant $b > 0$.

Similarly, we shall replace the discrete fields

$$\{\phi_j(\mathbf{r}(x,t),t), \ j \in \{1, \cdots, N\}\},$$

by a continuous one in s

$$\phi(\mathbf{r}(x,t),t,s)$$

and the real constants

$$\{\gamma_j, \alpha_{jk}, \ j,k \in \{1, \cdots N\}\}$$

by appropriate positive functions

$$\gamma(s) \text{ and } \alpha(s,p),$$

where again $s, p \in [0,b]$.

Thus, with such replacements we redefine the functional J, now denoting it by J_1, as

$$
\begin{aligned}
& J_1(\mathbf{v},\mathbf{r},\hat{\mathbf{r}},\phi,\rho,T,\lambda,\hat{P},E,E_1,N) \\
= \; & \frac{1}{2}\int_0^{t_f}\int_\Omega \rho\mathbf{v}(\mathbf{r}(x,t),t)\cdot\mathbf{v}(\mathbf{r}(x,t),t)\,dx\,dt \\
& +\frac{1}{2}\int_0^b\int_0^{t_f}\int_\Omega |\phi(\mathbf{r}(x,t),t,s)|^2\frac{d\hat{\mathbf{r}}(\mathbf{r}(x,t),t,s)}{dt}\cdot\frac{d\hat{\mathbf{r}}(\mathbf{r}(x,t),t,s)}{dt}\,dxdtds \\
& +\int_0^b\int_0^{t_f}\int_\Omega |\phi(\mathbf{r}(x,t),t,s)|^2\frac{d\hat{\mathbf{r}}(\mathbf{r}(x,t),t,s)}{dt}\cdot\frac{d\mathbf{r}(x,t)}{dt}\,dxdtds \\
& +\left\langle \lambda, \mathbf{v}(\mathbf{r}(x,t),t)-\frac{\partial\mathbf{r}(x,t)}{\partial t}\right\rangle_{L^2} \\
& -\left\langle \hat{P}, \frac{d\rho}{dt}+\rho\frac{\partial}{\partial x_j}\left(\frac{\partial X_j(x,t)}{\partial t}\right)\right\rangle_{L^2} \\
& -\int_0^{t_f}\int_\Omega \rho\mathbf{g}\cdot\mathbf{r}(x,t)\,dxdt \\
& +\int_0^{t_f}\int_\Omega \lambda_1\left(\rho C_v\frac{dT}{dt}-K\nabla^2T+P(\hat{P},\rho)\ \mathrm{div}\ \mathbf{v}-\rho\mathbf{g}\cdot\mathbf{v}\right)\,dxdt \\
& +\int_0^{t_f}\int_\Omega \lambda_2\,(C_v\,T \\
& -\frac{1}{2}\int_0^b |\phi(\mathbf{r}(x,t),t,s)|^2\frac{d\hat{\mathbf{r}}(\mathbf{r}(x,t),t,s)}{dt}\cdot\frac{d\hat{\mathbf{r}}(\mathbf{r}(x,t),t,s)}{dt}\,ds\Big)\,dxdt \\
& +\frac{1}{2}\int_0^b\gamma(s)\int_0^{t_f}\int_\Omega \nabla\phi(\mathbf{r}(x,t),t,s)\cdot\nabla\phi(\mathbf{r}(x,t),t,s)\,dsdxdt \\
& +\frac{1}{2}\int_0^b\gamma_0\int_0^{t_f}\int_\Omega \frac{d\phi(\mathbf{r}(x,t),t,s)}{dt}\frac{d\phi(\mathbf{r}(x,t),t,s)}{dt}\,ds\,dxdt \\
& +\int_0^b\int_0^b\int_0^{t_f}\int_\Omega\int_\Omega \frac{\alpha(s,p)\,|\phi(\mathbf{r}(x,t),t,s)|^2|\phi(\mathbf{r}(\tilde{x},t),t,p)|^2}{|\mathbf{r}(x,t)-\mathbf{r}(\tilde{x},t)|}\,dx\,d\tilde{x}\,dt\,dsdp \\
& -\int_0^b\int_0^{t_f} E(t,s)\left(\int_\Omega |\phi(\mathbf{r}(x,t,s),t)|^2\,dx-N(s,t)\right)\,dt\,ds \\
& +\int_0^{t_f} E_1(t)\left(\int_0^b N(s,t)\,ds-m_T\right)\,dt \\
& +\int_0^{t_f}\int_\Omega \lambda_3\left(\rho(\mathbf{r}(x,t),t)-\int_0^b |\phi(\mathbf{r}(x,t),t,s)|^2\,ds\right)\,dxdt, \qquad (2)
\end{aligned}
$$

where we recall the pressure field is defined by

$$P = P(\hat{P}, \rho) = \frac{d(\rho \hat{P})}{dt}$$

and m_T is the fixed total system mass.

Remark 3.1. *We consider variations in the spaces $C_c^\infty(\Omega \times [0, t_f])$ and $C_c^\infty(\Omega \times [0, t_f] \times [0, b])$ and their vectorial analogous ones, so that we obtain the related Euler-lagrange equations in a distributional sense on these domains $\Omega \times [0, t_f]$ and $\Omega \times [0, t_f] \times [0, b]$ not addressing the concerning boundary and initial conditions in this first work.*

Symbolically, the variation of J in $\mathbf{v}$ stands for

$$\rho \mathbf{v} + \lambda - \nabla(\lambda_1 P) - \lambda_1 \rho \mathbf{g} = \mathbf{0},$$

in $\Omega \times [0, t_f]$.

Moreover, the variation of J in $\mathbf{r}$ stands for

$$\begin{aligned}
\mathbf{0} &= \frac{dJ_1(\mathbf{v}, \mathbf{r}, \hat{\mathbf{r}}, \phi, \rho, T, \lambda, \hat{P}, E, E_1, N)}{d\mathbf{r}} \\
&= \frac{\partial J_1(\mathbf{v}, \mathbf{r}, \hat{\mathbf{r}}, \phi, \rho, T, \lambda, \hat{P}, E, E_1, N)}{\partial \mathbf{v}} \frac{\partial \mathbf{v}}{\partial \mathbf{r}} \\
&\quad + \frac{\partial J_1(\mathbf{v}, \mathbf{r}, \hat{\mathbf{r}}, \phi, \rho, T, \lambda, \hat{P}, E, E_1, N)}{\partial \rho} \frac{\partial \rho}{\partial \mathbf{r}} \\
&\quad + \frac{\partial J_1(\mathbf{v}, \mathbf{r}, \hat{\mathbf{r}}, \phi, \rho, T, \lambda, \hat{P}, E, E_1, N)}{\partial \hat{\mathbf{r}}} \frac{\partial \hat{\mathbf{r}}}{\partial \mathbf{r}} \\
&\quad + \frac{\partial J_1(\mathbf{v}, \mathbf{r}, \hat{\mathbf{r}}, \phi, \rho, T, \lambda, \hat{P}, E, E_1, N)}{\partial \phi} \frac{\partial \phi}{\partial \mathbf{r}} \\
&\quad + \frac{\partial J_1(\mathbf{v}, \mathbf{r}, \hat{\mathbf{r}}, \phi, \rho, T, \lambda, \hat{P}, E, E_1, N)}{\partial T} \frac{\partial T}{\partial \mathbf{r}} \\
&\quad + \frac{\partial J_1(\mathbf{v}, \mathbf{r}, \hat{\mathbf{r}}, \phi, \rho, T, \lambda, \hat{P}, E, E_1, N)}{\partial \mathbf{r}} \\
&= \frac{\partial J_1(\mathbf{v}, \mathbf{r}, \hat{\mathbf{r}}, \phi, \rho, T, \lambda, \hat{P}, E, E_1, N)}{\partial \mathbf{r}}.
\end{aligned} \tag{3}$$

In summary, we must have

$$\frac{\partial J_1(\mathbf{v}, \mathbf{r}, \hat{\mathbf{r}}, \phi, \rho, T, \lambda, \hat{P}, E, E_1, N)}{\partial \mathbf{r}} = \mathbf{0},$$

so that

$$\frac{\partial\lambda}{\partial t} - \frac{d}{dt}\left(\int_0^b |\phi(\mathbf{r}(x,t),t,s)|^2 \frac{d\hat{\mathbf{r}}(\mathbf{r}(x,t),t,s)}{dt}\, ds\right)$$
$$-\int_\Omega \int_0^b \int_0^b \frac{|\phi(\mathbf{r}(x,t),t,s)|^2 |\phi(\mathbf{r}(\tilde{x},t),t,p)|^2 (\mathbf{r}(x,t)-\mathbf{r}(\tilde{x},t))}{|\mathbf{r}(x,t)-\mathbf{r}(\tilde{x},t)|^3}\, d\tilde{x}dsdp$$
$$-\rho\mathbf{g} = \mathbf{0}, \tag{4}$$

in $\Omega \times [0, t_f]$.

Denoting

$$\hat{v} = \frac{d\hat{\mathbf{r}}(\mathbf{r}(x,t),t,s)}{dt}$$

and combining these last two variations, we obtain

$$\frac{d(\rho\mathbf{v})}{dt} + \frac{d\nabla(\rho\hat{P})}{dt} - \frac{d(\nabla(\lambda_1 P))}{dt} - \frac{d(\lambda_1\rho\mathbf{g})}{dt}$$
$$+\frac{d}{dt}\left(\int_0^b |\phi(\mathbf{r}(x,t),t,s)|^2 \hat{v}\, ds\right)$$
$$+\int_\Omega \int_0^b \int_0^b \frac{|\phi(\mathbf{r}(x,t),t,s)|^2 |\phi(\mathbf{r}(\tilde{x},t),t,p)|^2 (\mathbf{r}(x,t)-\mathbf{r}(\tilde{x},t))}{|\mathbf{r}(x,t)-\mathbf{r}(\tilde{x},t)|^3}\, d\tilde{x}dsdp$$
$$+\rho\mathbf{g} = \mathbf{0}, \tag{5}$$

in $\Omega \times [0, t_f]$, so that

$$\frac{\partial(\rho\mathbf{v})}{\partial t} + \frac{\partial(\rho\mathbf{v})}{\partial X_j}\frac{\partial X_j}{\partial t}$$
$$+\frac{d\nabla(\rho\hat{P})}{dt} - \frac{d(\nabla(\lambda_1 P))}{dt} - \frac{d(\lambda_1\rho\mathbf{g})}{dt}$$
$$+\frac{d}{dt}\left(\int_0^b |\phi(\mathbf{r}(x,t),t,s)|^2 \hat{v}\, ds\right)$$
$$+\int_\Omega \int_0^b \int_0^b \frac{|\phi(\mathbf{r}(x,t),t,s)|^2 |\phi(\mathbf{r}(\tilde{x},t),t,p)|^2 (\mathbf{r}(x,t)-\mathbf{r}(\tilde{x},t))}{|\mathbf{r}(x,t)-\mathbf{r}(\tilde{x},t)|^3}\, d\tilde{x}dsdp$$
$$+\rho\mathbf{g} = \mathbf{0}, \tag{6}$$

in $\Omega \times [0, t_f]$, that is,

$$\begin{aligned}
&\frac{\partial(\rho \mathbf{v})}{\partial t} + \frac{\partial(\rho \mathbf{v})}{\partial X_j} v_j \\
&+\nabla P - \frac{d(\nabla(\lambda_1 P))}{dt} - \frac{d(\lambda_1 \rho \mathbf{g})}{dt} \\
&+\frac{d}{dt}\left(\int_0^b |\phi(\mathbf{r}(x,t),t,s)|^2 \hat{v}\, ds\right) \\
&+\int_\Omega \int_0^b \int_0^b \frac{|\phi(\mathbf{r}(x,t),t,s)|^2 |\phi(\mathbf{r}(\tilde{x},t),t,p)|^2 (\mathbf{r}(x,t) - \mathbf{r}(\tilde{x},t))}{|\mathbf{r}(x,t) - \mathbf{r}(\tilde{x},t)|^3} d\tilde{x} ds dp \\
&+\rho \mathbf{g} = \mathbf{0},
\end{aligned} \tag{7}$$

in $\Omega \times [0, t_f]$,

To clarify the notation, we highlight that denoting

$$\rho \hat{P} = \rho(\mathbf{r}(x,t),t)\hat{P}(x,t) \equiv W(x,t),$$

we have

$$\frac{\partial(\nabla(\rho \hat{P}))}{\partial t} = \frac{\partial \nabla(W(x,t))}{\partial t} = \nabla\left(\frac{\partial W(x,t)}{\partial t}\right).$$

The variation of J in λ stands for

$$\mathbf{v} - \frac{\partial \mathbf{r}}{\partial t} = \mathbf{0}.$$

The variation of J in $\hat{P}$ stands for

$$\frac{d\rho}{dt} + \rho \frac{\partial}{\partial x_j}\left(\frac{\partial X_j(x,t)}{\partial t}\right) + \frac{d(\lambda_1 \operatorname{div} \mathbf{v})}{dt} \rho = 0, \text{ in } \Omega \times [0, t_f].$$

Denoting $v_j = \frac{\partial X_j(x,t)}{\partial t}$ this last equation stands for

$$\frac{d\rho}{dt} + \rho \frac{\partial(v_j)}{\partial x_j} + \frac{d(\lambda_1 \operatorname{div} \mathbf{v})}{dt} \rho = 0, \text{ in } \Omega \times [0, t_f].$$

Finally, the variation of J in ρ stands for

$$\frac{1}{2}\mathbf{v} \cdot \mathbf{v} + \frac{\partial \hat{P}}{\partial t} - \hat{P} \frac{\partial v_j}{\partial x_j} - \mathbf{g} \cdot \mathbf{r} - \lambda_1 C_v \frac{dT}{dt} - \lambda_1 \mathbf{g} \cdot \mathbf{v} + \hat{P} \frac{d(\lambda_1 \operatorname{div} \mathbf{v})}{dt} + \lambda_3 = 0,$$

in $\Omega \times [0, t_f]$.

Remark 3.2. *Recalling the pressure field is expressed by*

$$P = \frac{\partial(\rho \hat{P})}{\partial t}$$

we have obtained

$$\begin{aligned} & \frac{1}{2}\mathbf{v}\cdot\mathbf{v} + \frac{P}{\rho} - \frac{1}{\rho}\frac{d\rho}{dt}\hat{P} - \hat{P}\frac{\partial v_j}{\partial x_j} + gX_3(x,t) \\ & -\lambda_1 C_v \frac{dT}{dt} - \lambda_1 \mathbf{g}\cdot\mathbf{v} - \hat{P}\frac{d(\lambda_1\ div\ \mathbf{v})}{dt} + \lambda_3 \\ = \ & \frac{1}{2}\mathbf{v}\cdot\mathbf{v} + \frac{P}{\rho} + gX_3(x,t) - \lambda_1 C_v \frac{dT}{dt} - \lambda_1 \mathbf{g}\cdot\mathbf{v} + \lambda_3 = 0, \end{aligned} \tag{8}$$

in $\Omega \times [0, t_f]$.

In summary, we have obtained the following generalization of the Bernoulli equation

$$\frac{1}{2}\mathbf{v}\cdot\mathbf{v} + \frac{P}{\rho} + gX_3 - \lambda_1 C_v \frac{dT}{dt} - \lambda_1 \mathbf{g}\cdot\mathbf{v} + \lambda_3 = 0, \ \ in\ \Omega \times [0, t_f].$$

The variation of J_1 in ϕ stands for

$$\begin{aligned} & -\gamma_0 \frac{d^2\phi}{dt^2} - \gamma(s)\nabla^2\phi + \phi\frac{d\hat{\mathbf{r}}}{dt}\cdot\frac{d\hat{\mathbf{r}}}{dt} \\ & -\lambda_2\ \phi\frac{d\hat{\mathbf{r}}}{dt}\cdot\frac{d\hat{\mathbf{r}}}{dt} + 2\phi\frac{d\hat{\mathbf{r}}}{dt}\cdot\frac{d\mathbf{r}}{dt} \\ & + \int_\Omega\int_0^b \frac{|\phi(\mathbf{r}(x,t),t,s)||\phi(\mathbf{r}(\tilde{x},t),t,p)|^2}{|\mathbf{r}(x,t)-\mathbf{r}(\tilde{x},t)|^3} d\tilde{x}dp \\ & -E(t,s)\phi - 2\lambda_3\phi = 0, \ \text{in}\ \Omega\times[0,t_f]\times[0,b]. \end{aligned} \tag{9}$$

The variation of J_1 in $\hat{\mathbf{r}}$ stands for

$$-\frac{d(|\phi|^2\frac{d\hat{\mathbf{r}}}{dt})}{dt} + \frac{d(\lambda_2|\phi|^2\frac{d\hat{\mathbf{r}}}{dt})}{dt} - 2\frac{d(|\phi|^2\mathbf{v})}{dt} = \mathbf{0}$$

in $\Omega \times [0, t_f] \times [0, b]$.

The variation of J_1 in T stands for

$$-C_v\ \frac{d(\lambda_1\rho)}{dt} - K\nabla^2\lambda_1 + C_v\lambda_2 = 0, \ \text{in}\ \Omega\times[0,t_f].$$

The variation of J_1in λ_1 stands for

$$C_v\ \rho\ \frac{d\,T(\mathbf{r}(x,t),t)}{dt} - K\nabla^2 T - \rho\mathbf{g}\cdot\mathbf{v} + P\ \text{div}\ (\mathbf{v}) - \frac{\partial Q}{\partial t} = 0,$$

in $\Omega \times [0, t_f]$.

The variation of J_1 in λ_2 stands for

$$C_v\, T - \frac{1}{2}\int_0^b |\phi(\mathbf{r}(x,t),t,s)|^2 \frac{d\hat{\mathbf{r}}(\mathbf{r}(x,t),t,s)}{dt} \cdot \frac{d\hat{\mathbf{r}}(\mathbf{r}(x,t),t,s)}{dt}\, ds = 0,$$

in $\Omega \times [0, t_f]$.

The variation of J_1 in λ_3 stands for

$$\rho(\mathbf{r}(x,t),t) - \int_0^b |\phi(\mathbf{r}(x,t),t,s)|^2\, ds = 0, \text{ in } \Omega \times [0, t_f].$$

The variation of J_1 in N stands for

$$E(t,s) + E_1(t) = 0, \text{ in } [0, t_f] \times [0, b].$$

The variation of J_1 in E_1, stands for

$$\int_0^b N(s,t)\, ds - m_T = 0, \text{ in } [0, t_f].$$

Finally, the variation of J_1 in E, stands for

$$\int_\Omega |\phi(\mathbf{r}(x,t),t,s|^2\, dx - N(s,t) = 0, \text{ in } [0, t_f] \times [0, b].$$

Remark 3.3. *To clarify the notation, we firstly recall that*

$$\begin{aligned} \frac{dT(\mathbf{r}(x,t),t)}{dt} &= \frac{\partial T}{\partial t} + \frac{\partial T}{\partial X_j}\frac{\partial X_j(x,t)}{\partial t} \\ &= \frac{\partial T}{\partial t} + \frac{\partial T}{\partial X_j} v_j. \end{aligned} \tag{10}$$

Moreover, we also recall that generically for a scalar smooth function $\varphi = \varphi(\mathbf{r}(x,t),t)$, *we have*

$$\frac{\partial \varphi}{\partial x_j} = \frac{\partial \varphi}{\partial X_k}\frac{\partial X_k}{\partial x_j}.$$

Thus, denoting

$$\{A_{jk}\} = \left\{\frac{\partial X_k(x,t)}{\partial x_j}\right\}^{-1},$$

the equation

$$\begin{aligned}
&\frac{\partial(\rho\mathbf{v})}{\partial t}+\frac{\partial(\rho\mathbf{v})}{\partial X_j}v_j\\
&+\nabla P-\frac{d(\nabla(\lambda_1 P))}{dt}-\frac{d(\lambda_1\rho\mathbf{g})}{dt}+\frac{d}{dt}\left(\int_0^b|\phi(\mathbf{r}(x,t),t,s)|^2\hat{v}\right)\\
&+\int_\Omega\int_0^b\int_0^b\frac{|\phi(\mathbf{r}(x,t),t,s)|^2|\phi(\mathbf{r}(\tilde{x},t),t,p)|^2(\mathbf{r}(x,t)-\mathbf{r}(\tilde{x},t))}{|\mathbf{r}(x,t)-\mathbf{r}(\tilde{x},t)|^3}d\tilde{x}dsdp\\
&+\rho\mathbf{g}=\mathbf{0},
\end{aligned}\tag{11}$$

in $\Omega\times[0,t_f]$, *stands for*

$$\begin{aligned}
&\frac{\partial(\rho\mathbf{v})}{\partial t}+A_{jl}\frac{\partial(\rho\mathbf{v})}{\partial x_l}v_j\\
&+\nabla P-\frac{d(\nabla(\lambda_1 P))}{dt}-\frac{d(\lambda_1\rho\mathbf{g})}{dt}\\
&+\frac{d}{dt}\left(\int_0^b|\phi(\mathbf{r}(x,t),t,s)|^2\hat{v}ds\right)\\
&+\int_\Omega\int_0^b\int_0^b\frac{|\phi(\mathbf{r}(x,t),t,s)|^2|\phi(\mathbf{r}(\tilde{x},t),t,p)|^2(\mathbf{r}(x,t)-\mathbf{r}(\tilde{x},t))}{|\mathbf{r}(x,t)-\mathbf{r}(\tilde{x},t)|^3}d\tilde{x}dsdp\\
&+\rho\mathbf{g}=\mathbf{0},
\end{aligned}\tag{12}$$

in $\Omega\times[0,t_f]$.

Finally, with the Eulerian identification

$$X_k(x,t)\approx x_k$$

and denoting $x_3=z+c_1$, *we obtain*

$$\{A_{kj}\}\approx I_d$$

where I_d *denotes the identity matrix, so that we obtain the final format for such*

Eulerian equation:

$$\begin{aligned}
&\frac{\partial(\rho\mathbf{v})}{\partial t}+\frac{\partial(\rho\mathbf{v})}{\partial x_j}v_j\\
&+\nabla P-\frac{\partial(\nabla(\lambda_1 P))}{\partial t}-\frac{\partial(\nabla(\lambda_1 P))}{\partial x_j}v_j\\
&-\frac{\partial(\lambda_1\rho\mathbf{g})}{\partial t}-\frac{\partial(\lambda_1\rho\mathbf{g})}{\partial x_j}v_j\\
&+\frac{\partial}{\partial t}\left(\int_0^b\phi(x,t,s)^2\hat{v}ds\right)+\frac{\partial}{\partial x_j}\left(\int_0^b\phi(x,t,s)^2\hat{v}ds\right)v_j\\
&+\int_\Omega\int_0^b\int_0^b\frac{|\phi(x,t,s)|^2|\phi(\tilde{x},t,p)|^2(x-\tilde{x})}{|x-\tilde{x}|^3}d\tilde{x}dsdp\\
&+\rho\mathbf{g}=\mathbf{0},
\end{aligned}\tag{13}$$

in $\Omega\times[0,t_f]$.

In such a context, the variation of J *in* $\hat{P}$ *stands for*

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho v_j)}{\partial x_j}+\frac{\partial(\lambda_1\ div\ \mathbf{v})}{\partial t}\rho+\frac{\partial(\lambda_1\ div\ \mathbf{v})}{\partial x_j}v_j\rho=0,\ \ in\ \Omega\times[0,t_f].$$

The variation of J *in* ρ *stands for*

$$\frac{1}{2}\mathbf{v}\cdot\mathbf{v}+\frac{P}{\rho}+gx_3-\lambda_1 C_v\frac{\partial T}{\partial t}-\lambda_1 C_v\frac{\partial T}{\partial x_j}v_j+\lambda_1\mathbf{g}\cdot\mathbf{v}+\lambda_3=0,$$

in $\Omega\times[0,t_f]$.

Finally, for the remaining equations, similar remarks hold.

The objective of this section is complete.

4 A note on a more general relativistic context

In this section we address the modeling of a fluid motion in an Eulerian relativistic context.

Let $\Omega\subset\mathbb{R}^3$ be an open, bounded and connected set with a regular (Lipschitzian) boundary denoted by $\partial\Omega$.

At first we consider such a fluid in question as a particle system one with macroscopic position field $\mathbf{r}:\Omega\times[0,t_f]\to\mathbb{R}^4$, where

$$\mathbf{r}(x,t)=(c\,t,X_1(x,t),X_2(x,t),X_3(x,t)).$$

Here $t \in [0, t_f]$ denotes time and c denotes the speed of light.

For particles p_j with a state corresponding to the eigenvalue $E_j(t)$, we define the related position field by

$$\mathbf{r}_j(x,t) = \mathbf{r}(x,t) + \hat{\mathbf{r}}_j(\mathbf{r}(x,t)),\ \forall j \in \{1, \cdots, N\},$$

where the fields $\{\hat{\mathbf{r}}_j(\mathbf{r}(x,t))\}$ correspond to internal vibration degrees of freedom, related to internal energy and temperature concepts.

The corresponding densities fields are denoted by $|\phi_j(\mathbf{r}(x,t),t)|^2$ with corresponding relativistic densities

$$\frac{|\phi_j(\mathbf{r}(x,t),t)|^2}{\sqrt{1-\frac{v^2}{c^2}}},$$

where

$$v^2 = \left(\frac{\partial X_1(x,t)}{\partial t}\right)^2 + \left(\frac{\partial X_2(x,t)}{\partial t}\right)^2 + \left(\frac{\partial X_3(x,t)}{\partial t}\right)^2,$$

$$\frac{\partial \mathbf{r}(x,t)}{\partial t} \cdot \frac{\partial \mathbf{r}(x,t)}{\partial t} = -c^2 + v^2.$$

Here we have generically denoted

$$\mathbf{y} \cdot \mathbf{z} = -y_0 z_0 + y_1 z_1 + y_2 z_2 + y_3 z_3,$$

$\forall \mathbf{y} = (y_0, y_1, y_2, y_3),\ \mathbf{z} = (z_0, z_1, z_2, z_3) \in \mathbb{R}^4$.

The macroscopic density field at a rest is defined by

$$\rho_1(\mathbf{r}(x,t),t) = \sum_{j=1}^{N} |\phi_j(\mathbf{r}(x,t),t)|^2 \equiv |\phi|^2,$$

and the related relativistic density field is defined by

$$\rho(\mathbf{r}(x,t),t) = \sum_{j=1}^{N} \frac{|\phi_j(\mathbf{r}(x,t),t)|^2}{\sqrt{1-\frac{v^2}{c^2}}},$$

We define also a macroscopic system velocity field by

$$\mathbf{v}(\mathbf{r}(x,t),t)$$

which stands for the system velocity field at the position $\mathbf{r}(x,t)$ and time t.

Moreover, we set the following obvious constraint

$$\mathbf{v}(\mathbf{r}(x,t),t) = \frac{\partial \mathbf{r}(x,t)}{\partial t},\ \text{in } \Omega \times [0, t_f].$$

Finally, the non-constant scalar field of temperature is denoted by

$$T = T(\mathbf{r}(x,t),t),$$

where we define

$$C_v\, T(\mathbf{r}(x,t),t) = \frac{1}{2}\sum_{j=1}^{N} \frac{|\phi_j(\mathbf{r}(x,t),t)|^2\, d\hat{\mathbf{r}}_j(\mathbf{r}(x,t),t)}{\sqrt{1-\frac{v^2}{c^2}}\, dt} \cdot \frac{d\hat{\mathbf{r}}_j(\mathbf{r}(x,t),t)}{dt}.$$

Furthermore, following the approach in [16], we assume the following energy equation

$$C_v\rho\frac{1}{\sqrt{-g}}\frac{d(\sqrt{-g}\, T(\mathbf{r}(x,t),t))}{dt} - K\nabla^2 T - \rho\mathbf{g}\cdot\mathbf{v} + P\ \mathrm{div}\ (\mathbf{v}) - \frac{\partial Q}{\partial t} = 0,$$

in $\Omega \times [0,t_f]$, where as previously indicated,

$$T = T(\mathbf{r}(x,t),t)$$

is the temperature scalar field, $C_v > 0$, $K > 0$ are real constants and Q is a heat function.

Here, concerning the manifold in question, we have denoted

$$\nabla^2 T = \frac{1}{\sqrt{-g}}\left[\frac{\partial}{\partial x_i}\left(g^{ij}\frac{\partial T}{\partial x_j}\sqrt{-g}\right)\right],$$

where $x_0 = t$,

$$\mathbf{g}_k = \frac{\partial \mathbf{r}(x,t)}{\partial x_k},$$

$$g_{jk} = \mathbf{g}_j \cdot \mathbf{g}_k,\ \forall j,k \in \{0,1,2,3\},$$

$$\{g^{jk}\} = \{g_{jk}\}^{-1}$$

and

$$g = \det\{g_{jk}\}.$$

Moreover, also considering the manifold in question, we have denoted

$$\mathrm{div}\ \mathbf{v} = \frac{1}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\, v_j)}{\partial x_j}.$$

Finally, we assume the following mass conservation equation as a constraint

$$\frac{1}{\sqrt{-g}}\frac{d(\sqrt{-g}\,\rho)}{dt} + \rho\frac{1}{\sqrt{-g}}\frac{\partial(v_j\sqrt{-g})}{\partial x_j} = 0,\ \text{in}\ \Omega\times[0,t_f].$$

Concerning such a fluid motion modeling, neglecting the self interaction term and denoting generically $Y = C^1$, $Y_1 = C^2$ and $V = C_0^2$, we define the following variational formulation represented by the functional $J : Y \times V \times Y_1 \to \mathbb{R}$ already including the concerning Lagrange multipliers for the constraints, where

$$
\begin{aligned}
& J(\mathbf{v}, \mathbf{r}, \hat{\mathbf{r}}, \{\phi_j\}, \rho, T, \lambda, \hat{P}, E, \mu, N) \\
= \; & \frac{1}{2}\int_0^{t_f}\int_\Omega \rho \mathbf{v}(\mathbf{r}(x,t),t)\cdot\mathbf{v}(\mathbf{r}(x,t),t)\,\sqrt{-g}\,dx\,dt \\
& + \frac{1}{2}\sum_{j=1}^{N}\int_0^{t_f}\int_\Omega \frac{|\phi_j(\mathbf{r}(x,t),t)|^2\, d\hat{\mathbf{r}}_j(\mathbf{r}(x,t),t)}{\sqrt{1-\frac{v^2}{c^2}}\qquad dt}\cdot\frac{d\hat{\mathbf{r}}_j(\mathbf{r}(x,t),t)}{dt}\,\sqrt{-g}\,dxdt \\
& + \sum_{j=1}^{N}\int_0^{t_f}\int_\Omega \frac{|\phi_j(\mathbf{r}(x,t),t)|^2\, d\hat{\mathbf{r}}_j(\mathbf{r}(x,t),t)}{\sqrt{1-\frac{v^2}{c^2}}\qquad dt}\cdot\frac{d\mathbf{r}(x,t)}{dt}\,\sqrt{-g}\,dxdt \\
& + \left\langle \lambda, \mathbf{v}(\mathbf{r}(x,t),t) - \frac{\partial \mathbf{r}(x,t)}{\partial t}\right\rangle_{L^2} \\
& - \int_0^{t_f}\int_\Omega \hat{P}\left(\frac{1}{\sqrt{-g}}\frac{d(\sqrt{-g}\rho_1)}{dt} + \frac{\rho_1}{\sqrt{-g}}\frac{\partial(v_j\sqrt{-g})}{\partial x_j}\right)\sqrt{-g}\,dxdt \\
& - \int_0^{t_f}\int_\Omega \rho\mathbf{g}\cdot\mathbf{r}(x,t)\sqrt{-g}\,dxdt \\
& + \int_0^{t_f}\int_\Omega \lambda_1\left(\cancel{C_v\rho}\frac{1}{\sqrt{-g}}\frac{d(\sqrt{-g}\,T(\mathbf{r}(x,t),t))}{dt} - K\nabla^2 T - \rho\mathbf{g}\cdot\mathbf{v}\right. \\
& \left. + P\ \mathrm{div}\ (\mathbf{v}) - \frac{\partial Q}{\partial t}\right)\sqrt{-g}\,dxdt \\
& + \int_0^{t_f}\int_\Omega \lambda_2\left(C_v\,T - \frac{1}{2}\sum_{j=1}^{N}\frac{|\phi_j|^2\quad d\hat{\mathbf{r}}_j(\mathbf{r}(x,t),t)}{\sqrt{1-\frac{v^2}{c^2}}\qquad dt}\cdot\frac{d\hat{\mathbf{r}}_j(\mathbf{r}(x,t),t)}{dt}\right)\sqrt{-g}\,dxdt \\
& + \frac{\gamma}{2}\int_0^{t_f}\int_\Omega R\sqrt{-g}\,dxdt \\
& - \sum_{j=1}^{N}\int_0^{t_f} E_j(t)\left(\int_\Omega |\phi_j(\mathbf{r}(x,t),t)|^2\sqrt{-g}\,dx - N_j(t)m_p\right)dt \\
& - \int_0^{t_f}\mu(t)\left(N_0 - \sum_{j=1}^{N}N_j(t)\right)dt \\
& + \sum_{j=1}^{N}\int_0^{t_f}\int_\Omega T\,\frac{N_j(t)}{N_0}\ln\left(\frac{N_j(t)}{N_0\,e}\right)\sqrt{-g}\,dxdt \\
& + \int_\Omega \lambda_3\left(\rho(\mathbf{r}(x,t),t) - \sum_{j=1}^{N}\frac{|\phi_j(\mathbf{r}(x,t),t)|^2}{\sqrt{1-\frac{v^2}{c^2}}}\right)\sqrt{-g}\,dxdt. \qquad (14)
\end{aligned}
$$

$$R = g^{jk} R_{jk},$$
$$R_{jk} = R_e[R^i_{jik}(\phi, g)],$$

and

$$\begin{aligned} R^l_{ijk}(\phi, g) &= |\phi|^2 \hat{R}^l_{ijk} \\ &\quad + \frac{\partial \phi}{\partial x_i} \frac{\partial \phi^*}{\partial x_j} \delta_{kl} \\ &\quad + \frac{\partial \phi}{\partial x_j} \phi^* \Gamma^l_{ik}. \end{aligned} \tag{15}$$

Finally, the Riemann curvature tensor stands for

$$\hat{R}^l_{ijk} = \frac{\partial \Gamma^l_{jk}}{\partial x_i} - \frac{\partial \Gamma^l_{ik}}{\partial x_j} + \Gamma^p_{jk} \Gamma^l_{ip} - \Gamma^p_{ik} \Gamma^l_{jp}. \tag{16}$$

and for the Levi-Civita connection,

$$\Gamma^i_{jk} = g^{il} \left(\frac{\partial g_{kl}}{\partial x_j} + \frac{\partial g_{jl}}{\partial x_k} - \frac{\partial g_{jk}}{\partial x_l} \right), \ \forall i, j, k, l \in \{0, 1, 2, 3\}.$$

where the pressure field P is defined by

$$P = P(\hat{P}, \rho) = \frac{d(\hat{P}\rho)}{dt},$$
$$\gamma_j > 0, \ \forall j \in \{1, \cdots, N\}$$

and $\gamma > 0$ are appropriate real constants.

Finally, the entropy differential is defined by

$$dS(x, t) = -\sum_{j=1}^{N} \frac{N_j(t)}{N_0} \ln \left(\frac{N_j(t)}{N_0 \, e} \right) \sqrt{-g} \, dx dt,$$

so that

$$\int_0^{t_f} \int_\Omega T \, dS = -\sum_{j=1}^{N} \int_0^{t_f} \int_\Omega T(\mathbf{r}(x, t), t) \frac{N_j(t)}{N_0} \ln \left(\frac{N_j(t)}{N_0 \, e} \right) \sqrt{-g} \, dx dt \, .$$

Here N_0 denotes a fixed total number of particles and $N_j(t)$ the number of particles at the state corresponding to $E_j(t), \forall j \in \{1, \cdots, N\}$, so that

$$N_0 = \sum_{j=1}^{N} N_j(t), \text{ in } [0, t_f].$$

Remark 4.1. *Observe that the variation of J in N_j give us*

$$-E_j(t)m_p - \mu(t) - \int_\Omega T\sqrt{-g}dx\frac{\partial}{\partial N_j(t)}\left(\frac{N_j(t)}{N_0}\ln\left(\frac{N_j(t)}{N_0}\right) - \frac{N_j(t)}{N_0}\right),$$

so that

$$-E_j(t)m_p - \mu(t) - \int_\Omega T\sqrt{-g}\,dx\left(\frac{1}{N_0}\ln\left(\frac{N_j(t)}{N_0}\right) + \frac{1}{N_0} - \frac{1}{N_0}\right),$$

that is,

$$-E_j(t)m_p - \mu(t) - \int_\Omega T\sqrt{-g}\,dx\frac{1}{N_0}\ln\left(\frac{N_j(t)}{N_0}\right),$$

and thus

$$\frac{N_j(t)}{N_0} = e^{N_0\left(\frac{-E_j(t)m_p-\mu(t)}{\int_\Omega T\sqrt{-g}\,dx}\right)},\ \textit{in}\ [0,t_f].$$

Such a distribution in $N_j(t)$ obtained is similar, in some sense, to the well known Gibbs one obtained in statistical physics.

5 Conclusion

In this article we have developed variational formulations for modeling a fluid motion in an Eulerian context including a scalar temperature field.

In this second work we have addressed a relativistic case as well. We highlight the number N is in general very large so that tools of statistical physics may be a possibility to address such a high magnitude for N. Anyway, we believe the internal energy variables may be appropriately modeled in such a more general quantum relativistic context.

Finally, for the second non-relativistic model, we emphasize to have obtained an additional term involving the Lagrange multiplier λ_1 in the continuity equation. We believe such a mentioned part results from the effect of the internal energy variables on the mass conservation equation.

References

[1] P. Constantin, C. Foias, *Navier-Stokes equations,* University of Chicago Press, Chicago, 1989.

[2] C. De Lellis, E. Brué, D. Albritton, M. Colombo, V. Giri, M. Janisch and H. Kwon, *Instability and Non-uniqueness for the 2D Euler Equations,* after M. Vishik (AMS-219) Volume 219 in the series Annals of Mathematics Studies, Princeton University Press, New Jersey, 2024. https://doi.org/10.1515/9780691257846

[3] F. Botelho, *Variational Formulations for the Euler System in Fluid Mechanics and Related Models.* Authorea. July 31, 2024. DOI: 10.22541/au.172243894.42056083/v1

[4] M. Hamouda, D. Han, C-Y. Jung, R. Temam, *Boundary layers for the 3D primitive equations in a cube: the zero-mode,* Journal of Applied Analysis and Computation. 2018; 8, No. 3, (873-889), DOI:10.11948/2018.873.

[5] A. Giorgini, A. Miranville, R. Temam, *Uniqueness and regularity for the Navier-Stokes-Cahn-Hilliard system,* SIAM J. of Mathematical Analysis (SIMA).2019; 51, 3, (2535-2574), https://doi.org/10.1137/18M1223459.

[6] C. Foias, R.M. Rosa, R. Temam R, *Properties of stationary statistical solutions of the three-dimensional Navier-Stokes equations,* J. of Dynamics and Differential Equations, Special issue in memory of George Sell. 2019; 31, 3, 1689-1741, https://doi.org/10.1007/s10884-018-9719-2.

[7] R. Temam, *Navier-Stokes Equations,* AMS Chelsea, reprint (2001).

[8] J.C. Strikwerda, *finite difference cchemes and partial differential equations*, second edition Philadelphia, SIAM, 2004.

[9] F.S. Botelho, *Approximate numerical procedures for the Navier-Stokes system through the generalized method of lines,* Nonlinear Engineering, vol. 13, no. 1, 2024, pp. 20220376. https://doi.org/10.1515/nleng-2022-0376

[10] F. Botelho, *Existence of solution for the Ginzburg-Landau system, a related optimal control problem and its computation by the generalized method of lines*, Applied Mathematics and Computation, 2012; 218 (11976-11989).

[11] F.S. Botelho, *An approximate proximal numerical procedure concerning the generalized method of lines.* Mathematics 2022, 10(16), 2950; https://doi.org/10.3390/math10162950.

[12] F. Botelho, *Functional Analysis and Applied Optimization in Banach Spaces*, (Springer Switzerland, 2014).

[13] F.S. Botelho, *Functional Analysis, Calculus of Variations and Numerical Methods for Models in Physics and Engineering,* CRC Taylor and Francis, Florida, 2020.

[14] F.S. Botelho, *Advanced Calculus and its Applications in Variational Quantum Mechanics and Relativity Theory,* CRC Taylor and Francis, Florida, 2021.

[15] R.A. Adams RA and J.F. Fournier, *Sobolev Spaces*, 2nd edn. (Elsevier, New York, 2003).

[16] J.C. Tannehill, D.A. Anderson, R.H. Pletcher, *Computational Fluid Mechanics and Heat Transfer,* 2nd edn., Taylor and Francis, Philadelphia, PA, 1997.

HADRONIC JOURNAL 48, 147 - 183 (2025)
DOI 10.29083/HJ.48.02.2025/SC147

TOWARDS AN EQUATION OF EVERYTHING IN THE THREE-DIMENSIONAL DYNAMIC QUANTUM VACUUM

Davide Fiscaletti
SpaceLife Institute – Via Roncaglia 35
S. Lorenzo in Campo (PU), Italy
spacelife.institute@gmail.com

Received February 24, 2025

Abstract

An equation of everything in a model of a three-dimensional dynamic quantum vacuum defined by a variable energy density and by a generalized Compton wavelength which provides a unified treatment of microphysics and macrophysics, is introduced. The consequences of this equation in the different regimes and as regards the interpretation of time as numerical order of material change are explored. Finally, a new solution of the paradoxes connected to black hole complementarity and information is proposed.

Keywords: equation of everything, three-dimensional dynamic quantum vacuum, generalized Compton wavelength, path integrals, numerical order of material change, black holes.

Copyright © 2025 by Hadronic Press, Inc., Palm Harbor, FL 34684, USA

1. Introduction

The Holy Graal of theoretical physics lies in the search of a theory encompassing all matter and forces, of a set of equations capable of describing all phenomena that have been observed and, in particular, that provides a synthesis of quantum physics and general relativity and a unified treatment of the four fundamental interactions. Several paths have been developed in order to address these problems, which go from the various approaches of superstring theory to loop quantum gravity to other less known approaches (see, for example, [1-10]).

The search of a theory of everything, if originally seemed to be seen as the consequence of the implementation of a reductionist approach, however, in more recent times, by virtue of the apparent impossibility of incorporating gravity in the context of workable quantum field theories and of recognizing it as a full member of the Standard Model, has assumed a new status, represented by the development of the so-called effective theories [11-21]. According to this line of research, one can make a separation between the scale we are working and the unknown physics regarding the much higher energies, thus arriving to obtain predictions at present energies without making unwarranted assumptions about what occurs at high energies. So, if the present Standard Model is an effective theory which breaks down at the TeV scale, where Higgs self-interactions would otherwise become unnaturally large, general relativity can be perhaps considered as the best example of an effective field theory since it involves very small quantum corrections, overlaps with quantum physics in regions where the latter is relevant and shows no hint of a breakdown before the Planck scale. Thanks to the development of the quantum techniques in the last 30 years, we have today a theory of Quantum General Relativity which treats general relativity, within various limits, as an effective field theory and is defined by a path integral over the dynamical degrees of freedom guided by a local Lagrangian.

The crucial idea of an effective field theory is that the low energy degrees of freedom organize themselves as quantum fields, ruled by a local Lagrangian, in general containing non-renormalizable terms suppressed by powers of a heavy scale, in such a way that one can make predictions without knowledge of the full high energy theory. In this picture, by investigating general relativity as a quantum field theory by means of the tools of effective field theory, in the region of validity of the effective field theory, it can be studied in perturbation theory and the quantum corrections are small, thus making it as the best perturbative theory ever, that leads to relevant lessons for the development of theoretical physics. In particular, in the Core Theory developed by Sean Carroll, which is an effective field theory that contains the well-known Standard Model of particle physics, but also quantum general relativity in the weak-field limit, the equation that underlies all the phenomena of our everyday experience is the following:

$$A = \int_{k<\Lambda}[Dg][DA][D\Psi][D\phi]\, exp\left\{\frac{i}{\hbar}\int d^4x\sqrt{-g}\left(\frac{c^4}{8\pi G}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}D_{\mu}\psi + \left|D_{\mu}\phi\right|^2 - V(\phi)\right) + \left(\bar{\psi}_L^i Y_{ij}\phi\psi_R^i + h.c.\right) + \sum_a \mathcal{O}^{(a)}(\Lambda)\right\} \quad (1)$$

where R is gravity, $F_{\mu\nu}F^{\mu\nu}$ are the gauge fields of strong, weak and electromagnetic interactions, $i\bar{\psi}\gamma^{\mu}D_{\mu}\psi$ is the kinetic energy of the fermion fields, $\left|D_{\mu}\phi\right|^2$ is the kinetic energy of the Higgs, $V(\phi)$ is the Higgs potential, $\bar{\psi}_L^i Y_{ij}\phi\psi_R^i$ is the Higgs-fermion interaction, the parameter $k < \Lambda$ is the cutoff in the overall path integral (which is of the order $10^{11}eV$) and the term $\sum_a \mathcal{O}^{(a)}(\Lambda)$ represents an infinite series of higher-order terms, each of which in general will be suppressed by powers of the cutoff, guaranteeing that physical predictions are independent of the cutoff value. According to equation (1), the domain of applicability of the Core Theory – and thus of an effective field theory – corresponds to energy and momenta that are all below the cutoff scale Λ in the sense that in this regime the theory is accurate [20, 21].

Although the dynamics described by effective field theories, such as the one summarized in equation (1) of the Core Theory, are well-defined, quantitative, as well as experimentally tested in a wide variety of contexts, however, it must emphasized that an essential feature of the structure of effective theories across physics lies in their intrinsic empirical limitations, i.e. the fact that they meet their own predictive failure at some scale. In this regard, the most straightforward realist explanation of this predictive pattern is to consider effective theories to accurately represent limited parts of the world [22].

In this paper, we want to throw new light as regards the problem of the search of an equation of everything which embeds the different regimes of physics, by going beyond the ideas subtending effective field theories and, in particular, by going beyond the equation (1) of Carroll's Core Theory and, at the same time, providing a new key of reading of the processes with respect to the well known reductionist approaches of quantum gravity such as superstring theory and loop quantum gravity. Here, our purpose is to suggest an alternative way to formulate an equation of everything, which is based on a model of a three-dimensional dynamic quantum vacuum (3D DQV), proposed by the author in recent papers [23-32], which postulates the existence of an underlying variable quantum vacuum energy density and, by starting from a peculiar specific form of generalized uncertainty relations, leads to define a universal scale that provides a unified treatment of macroscopic regime of black holes and quantum regime of elementary particles. We will see how the activity of the variable quantum vacuum energy density at this universal scale provide important lessons as regards the foundations and the development of theoretical

physics, which throw new light in the unifying description of the different processes of physics.

The paper is organized as follows. In section 2, we review the fundamental features of the model of the 3D DQV, which go to constitute important ingredients of the equation of everything. In section 3 we formulate our equation of everything in the 3D DQV and we derive its consequences in the different regimes, the dynamics laws for matter and gravity and the laws for the boundary condition of the universe. In section 4, we explore our interpretation of time in the theory of everything of the 3D DQV. In section 5 we suggest new perspectives of solution to some of the paradoxes connected to black holes. Finally, in section 6 we summarize the main results of the paper and we conclude by exploring the perspectives of the approach based on this equation of everything of the 3D DQV as regards the problem of finding a relation between the theory and the experience.

2. The ingredients of the equation of everything in the three-dimensional dynamic quantum vacuum

As regards the challenge of contemporary theoretical physics directed towards a unified description of quantum phenomena and gravity, an important research line that has received attention in the last thirty years is represented by the so-called generalized uncertainty principle, namely the idea that, at the quantum-gravity scale, the Heisenberg uncertainty relations should be modified in order to take into account the existence of a minimal length. Different forms of the generalized uncertainty principle have been explored in different domains, ranging from string theory and loop quantum gravity, to deformed special relativity, black hole physics, non-commutative geometry and asymptotically safe gravity [33-42].

In the model of the 3D DQV, we assume that the geometry of the background is expressed by the following peculiar specific form of generalized uncertainty relations, which are valid at the Planck scale:

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left(1 + \beta l_p^2 \frac{\Delta \rho_{qvE}^2 V^2}{\hbar^2 c^2}\right) \quad (2)$$

In equation (2) $\Delta\rho_{qvE}$ is the variable energy density of the vacuum corresponding to the appearance of a material particle of mass m in a volume V, defined as

$$\Delta\rho_{qvE} \equiv \rho_{pE} - \rho_{qvE} = \frac{mc^2}{V} \quad (3),$$

where

$$\rho_{PE} = \frac{m_p \cdot c^2}{l_p^3} \quad (4)$$

is Planck's energy density (m_P being Planck's mass, c the light speed and l_p Planck's length) intended as the ground state of the 3D DQV, β is a fluctuating dimensionless parameter which expresses the fact that here one deals with quantum vacuum fluctuations, associated with virtual particles, that fix the minimal length scale only on average, $\hbar$ is Planck's reduced constant and the term $\frac{\hbar}{2}\beta l_p^2 \frac{\Delta\rho_{qvE^V}^2}{\hbar^2 c^2}$ measures the degree of violations of the Heisenberg uncertainty relations at scales that approach the Planck scale. The generalized uncertainty relations (2) lead to a suggestive unification of the quantum regime of elementary particles and the macroscopic regime of black holes in terms of a generalized Compton wavelength which provides a unified expression for the Compton wavelength and Schwarzschild radius, given by relation

$$R'_C \approx R'_S = \sqrt{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2} \quad (5).$$

Equation (5) implies that the generalized Compton wavelength (5) tends to the standard value of the Schwarzschild radius for $\frac{\Delta\rho_{qvE^V}}{c^2} \gg M_{Pl}$ where M_{Pl} is the Planck mass, while tends to the quantum regime (namely to the standard value of the Compton wavelength associated to an elementary particle) for $\frac{\Delta\rho_{qvE^V}}{c^2} \ll M_{Pl}$. The physical meaning of equation (5) is that the origin of elementary particles of the quantum world and of material objects and black holes of macroscopic regime, is represented by elementary objects of the Planck scale, namely sub-Planckian black holes with a size of order of their Compton wavelength, which are generated by the geometrodynamic properties of the variable quantum vacuum energy density ultimately associated to the foam of the virtual particles of the 3D DQV [31, 32].

In the microscopic structure of the 3D DQV defined by the generalized Compton wavelength (5), the activity of the sub-Planckian black holes of the underlying variable energy density generates a deformation of the geometrical properties of the background expressed by a quantum-modified Schwarzschild metric which has the following form

$$ds^2 = F(r)c^2dt^2 - F(r)^{-1}dr^2 - r^2d\Omega^2 \quad (6)$$

where

$$F(r) = 1 - \frac{\sqrt{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE^V}}{\hbar c}\right)^2}}{r} + \varepsilon \frac{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE^V}}{\hbar c}\right)^2}{4r^2} \quad (7)$$

and $d\Omega^2 = d\theta^2 + sin^2\theta d\phi^2$ is the metric on the 3-sphere and ε is a dimensionless deformation parameter of the metric that, in analogy to the treatment made in [43], satisfies the constraint $|\varepsilon| \leq 1$. The quantum-modified metric (6) determines corrections in the values of the global deflection angle of light, the total precession of the orbit of a planet, the time dilation and the

gravitational red-shift with respect to the corresponding formulations predicted by standard general relativity, leading to general formulas which allow the standard corresponding general relativistic relations to be recovered under appropriate limits [44].

Moreover, each micro-region of the universe is described by a wave-function Ψ (which is a function of the scale factor a and of the scalar fields φ associated with the quantum vacuum energy density fluctuations as well as the polarization of the vacuum), that satisfies a peculiar form of Wheeler-DeWitt (WDW) equation, intended as an equation of background, a real "equation of everything" where all the possibilities of the physical world are written in timeless form:

$$\ddot{\Psi} + \frac{k\left(\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE}V}{\hbar\ c} - \right)^2\right)^{3/2} \Delta\rho_{qvE}}{ac^2\sqrt{a}}\dot{\Psi} + \omega^2(a)\Psi = 0 \quad (8).$$

In equation (8), one has the following quantities: $\dot{\Psi} = \frac{\partial\Psi}{\partial a}$, α is the fine-structure constant, ω is the mode frequency linked with the scale factor and, finally, k is an adimensional parameter corresponding to the size of the condensate of virtual sub-particles of the 3D DQV, namely represents a sort of effective parameter of density of the virtual particles of the medium [45]. By invoking the third quantization formalism, the wave function Ψ of the universe satisfying WDW equation (8) is promoted to the role of operator, which can be expressed as

$$\hat{\Psi}_{\pm} = \frac{V\Delta\rho_{qvE}}{\sqrt{2\pi\hbar l_p^2}} \int d\rho \left(e^{\pm i\frac{V\Delta\rho_{qvE}}{\hbar l_p^2}\varphi} \Psi_{\Delta\rho_{qvE}}(a)\hat{b}_{\Delta\rho_{qvE}} + e^{\mp i\frac{V\Delta\rho_{qvE}}{\hbar l_p^2}\varphi} \Psi^*_{\Delta\rho_{qvE}}(a)\hat{b}^{\dagger}_{\Delta\rho_{qvE}} \right) \quad (9)$$

where $\hat{b}_{\Delta\rho_{qvE}}$ and $\hat{b}^{\dagger}_{\Delta\rho_{qvE}}$ are the annihilation and creation operators which annihilate and create respectively micro-universes, associated with corresponding changes of the quantum vacuum energy density, and satisfy the "quantum Boltzmann statistics" that consists of a q deformation of the commutation relations of the oscillators, given by the following relation

$$\hat{b}_k\hat{b}^{\dagger}_l - q\hat{b}^{\dagger}_l\hat{b}_k = \delta_{kl} \quad (10)$$

and the cases $q = \pm 1$ correspond to bosons and fermions.

The wave function Ψ of a micro-universe satisfying the generalized WDW equation (7) gives origin to the manifestation of a corresponding particle if the information in a cell of the 3D DQV, that is defined by the following relation:

$$I = \frac{A}{l_p^2} \quad (11)$$

where

$$A = k\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2\right] \quad (12)$$

is the area of the cell, satisfies relation $l^3 l_p \approx 10^{-13} cm$, namely

$$\frac{k^3}{l_p^5}\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2\right]^3 \approx 10^{-13} cm \quad (13).$$

This means that one can choose opportunely the parameter k defining the size of the condensate of the virtual sub-particles of the medium in order to obtain the appearance of the desired particle of the Standard Model with its usual properties and, therefore, that the chronon scale of ordinary Standard Model particles can be interpreted as the emergence scale of processes of collective organization of opportune condensates of the virtual sub-particles of the 3D DQV which take place at the generalized Compton wavelength.

On the other hand, also the Higgs mass can be seen as the upper emergent manifestation resulting from the activity of the quantum vacuum energy density fluctuations at the generalized Compton wavelength [46]. The Mexican hat potential that in 3D DQV geometry plays the role of the Higgs mechanism is given by relation

$$V(\psi) = \frac{1}{4}\left[\frac{\xi^2}{2\pi} k\omega m|\Psi|^2\right]^{1/2}\left(\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2\right)^{-9/2} +$$
$$\left[\frac{\xi^2}{2\pi} k\omega m|\Psi|^2\right]^{1/2}\left(\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2\right)^{-3/2} h^2 +$$
$$\left[\frac{\xi^2}{2\pi} k\omega m|\Psi|^2\right]^{1/2} h^3 + \frac{1}{4}\left[\frac{\xi^2}{2\pi} k\omega m|\Psi|^2\right]^{1/2}\left(\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2 +$$
$$\left(\beta l_p^2 \frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2\right)^{3/2} h^4 \quad (14)$$

and the Higgs doublet is

$$\phi = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ \frac{1}{k\left(\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2\right)^{3/2} + h} \end{bmatrix} \quad (15)$$

where h denotes the excitations of the 3D DQV from the vacuum expectation value ε which is related to the rest mass of the W boson as

$$\varepsilon = \frac{\sqrt{2}c^2}{g} M_W \quad (16).$$

As regards the interaction of the Higgs with the gauge bosons, one can express it via the following equation

$$\left|\left(i\frac{g}{2}\sigma_i W_\mu^i + i\frac{g'}{2}B_\mu\right)\Psi\right|^2 = \frac{1}{8}\left[\frac{1}{k\left(\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2\right)^{3/2}}+h\right]^2 \times$$

$$\left[g^2\left(W_\mu^1\right)^2 + g^2\left(W_\mu^2\right)^2 + \left(-gW_\mu^3 + g'B_\mu\right)^2\right] \quad (17)$$

where g, g' are the gauge couplings. In the light of relation (17), one finds that the masses of the vector bosons are the following:

$$M_W = \frac{g}{2k_W\left(\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2\right)^{3/2}} \quad (18)$$

and

$$M_Z = \frac{\sqrt{g^2+g'^2}}{2k_Z\left(\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2\right)^{3/2}} \quad (19),$$

where k_W is the constant associated to the condensate of virtual sub-particles of the vacuum corresponding to the appearance of a W boson and k_Z is the constant associated to the condensate of virtual sub-particles of the vacuum corresponding to the appearance of a Z boson. In this way, the weak mixing angle expressing the relationship between the masses of W and Z bosons in the Standard Model, in the approach of the 3D DQV becomes

$$\cos\theta_W = \frac{gk_Z}{k_W\sqrt{g^2+g'^2}} \quad (20).$$

Finally, one can also find a new formulation for the Fermi constant

$$G_F = \frac{\sqrt{2}}{4}{k_W}^2\left(\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2\right)^3 \quad (21)$$

and here, by taking account of the numerical experimental value of the Fermi constant, one directly obtains the following constraint regarding the electroweak scale:

$$\frac{1}{k\left(\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2\right)^{3/2}} \sim 246\ GeV \quad (22).$$

As regards the treatment of the TeV scale of strong interactions, taking account of the results of [32, 46], a spontaneous dynamical scale symmetry breaking can be obtained by considering the mean-field approximated effective Lagrangian of the form

$$\mathcal{L}_{MFA} = \left([\partial^\mu s_R]^+\partial_\mu s_R\right) - \varepsilon^2(s_R{}^+ s_R) - \lambda_C(H^+H)^2 + N_f\left(N_f\lambda_S + \lambda_S{}'\right)f^2 + \frac{\lambda_S{}'}{2}(\phi^a)^2 - 2\lambda_S{}'\phi^a\left(s_R{}^+ t_{ij}^a s_R\right) \quad (23).$$

In equation (23) s_R is a scalar field depending on the energy fluctuations of the vacuum, $f = ({s_R}^+ s_R)/N_f$, H is the Higgs doublet field which defined by equation (12), $\phi^a = 2\left({s_R}^+ t_{ij}^a s_R\right)$ are auxiliary fields with t_{ij}^a generators of the flavour $SU(N_f)$ transformation and $\varepsilon = k\Delta\rho_{qvE}$ is a "constituent" scalar variable quantum vacuum energy of the hidden sector of the strong interactions – which is determined by opportune collective excitations of the virtual particles of the 3D DQV – given by relation

$$\varepsilon^2 = 2\left(N_f\lambda_S + \lambda_S{}'\right)f - \lambda_{SC}(H^+H) \quad (24).$$

Here, if one sets $\phi^a = 0$ in the mean-field approximation lagrangian, one can obtain the effective potential

$$V_{MFA} = k^2\frac{{M_P}^2c^4}{V^2}({\bar{s}_R}^+\bar{s}_R) + \lambda_C(H^+H)^2 - N_f\left(N_f\lambda_S + \lambda_S{}'\right)f^2 + \frac{N_C N_f\, k^4 {M_P}^4 c^8}{32\pi^2 \quad V^4} ln\frac{{M_P}^2c^4}{V^2\Lambda_C^2} \quad (25)$$

where $\bar{s}_R$ is the background field of the scalar field s_R. The effective potential (25), by applying the dimensional regularization and the $\overline{MS}$ scheme to eliminate a ultraviolet divergence, can be considered as the real mediating entity that is responsible of the spontaneous symmetry breaking occurring at the TeV scale, by leading directly to the following expressions for the changes of the vacuum energy associated to the collective excitations of the virtual particles of the 3D DQV, and the Higgs mass:

$$\left\langle k^2\frac{{M_P}^2c^4}{V^2}\right\rangle = \frac{4N_f\lambda_C\lambda_S - N_f\lambda_{SC}^2 + 4\lambda_C\lambda_{S'}}{2\lambda_C}\langle({s_R}^+ s_R)/N_f\rangle \quad (26)$$

$$M_h^2 \cong 2N_f\lambda_{SC}\langle({s_R}^+ s_R)/N_f\rangle \quad (27)$$

where here a small λ_{SC} is assumed. On the basis of equations (26) and (27), the variable quantum vacuum energy, the scalar field s_R depending on the energy fluctuations of the dissipative vacuum, as well as the scalar couplings λ_{SC}, λ_C, λ_S associated with the vacuum and with the singlet field depending on the quantum vacuum energy density fluctuations, can be considered the ultimate parameters which are responsible of the generation of the action of the Higgs boson in the high-energy regime. In other words, it is the interplay of opportune fluctuations of the energy of the 3D DQV which indeed determines the action of the Higgs boson generating a spontaneous symmetry breaking at the TeV scale.

3. The equation of everything and its predictions in the different regimes

In the light of all the results, mentioned in the previous chapter, of the model of the 3D DQV, as regards the quantum-modified metric associated with the generalized Compton wavelength, the condensate of the virtual sub-particles of the vacuum corresponding to the appearance of the particles of the Standard Model, as well as the interplay of specific fluctuations of the quantum vacuum

energy density mimicking the action of the Higgs boson, one can introduce the following equation of everything in the three-dimensional quantum vacuum model characterized by the unification scale represented by the generalized Compton wavelength:

$$W = \int [Dg][D\psi][D\Psi][D\phi]\, exp\left\{\frac{i}{\hbar}\int\left(\left[1-\frac{\sqrt{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE^V}}{\hbar c}\right)^2}}{r}+\varepsilon\frac{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE^V}}{\hbar c}\right)^2}{4r^2}\right]c^2dt^2-\left[1-\frac{\sqrt{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE^V}}{\hbar c}\right)^2}}{r}+\varepsilon\frac{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE^V}}{\hbar c}\right)^2}{4r^2}\right]^{-1}dr^2-r^2d\Omega^2\right)\sqrt{-g}\left(\frac{c^4}{8\pi G}R-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+i\bar{\psi}\gamma^\mu D_\mu\psi+\left|D_\mu\phi\right|^2-\left(\frac{1}{4}\left[\frac{\xi^2}{2\pi}k\omega m|\Psi|^2\right]^{1/2}\left(\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2\right)^{-9/2}+\left[\frac{\xi^2}{2\pi}k\omega m|\Psi|^2\right]^{1/2}\left(\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2\right)^{-3/2}h^2+\left[\frac{\xi^2}{2\pi}k\omega m|\Psi|^2\right]^{1/2}h^3+\frac{1}{4}\left[\frac{\xi^2}{2\pi}k\omega m|\Psi|^2\right]^{1/2}\left(\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2\right)^{3/2}h^4\right)\right)+\left(\bar{\psi}_L^i Y_{ij}\phi\psi_R^i\right)\right\}$$

(28).

In equation (28) R is the scalar curvature characterizing the region under consideration, g is the gravitational field, $F_{\mu\nu}F^{\mu\nu}$ are the gauge fields of the strong, weak and electromagnetic interactions, $\bar{\psi}\gamma^\mu D_\mu\psi$ is the kinetic energy of the fermion fields, γ^μ are the usual Dirac matrices, $\left|D_\mu\phi\right|^2$ is the kinetic energy of the Higgs,

$$\frac{1}{4}\left[\frac{\xi^2}{2\pi}k\omega m|\Psi|^2\right]^{1/2}\left(\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2\right)^{-9/2}+$$

$$\left[\frac{\xi^2}{2\pi}k\omega m|\Psi|^2\right]^{1/2}\left(\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2\right)^{-3/2}h^2+$$

$$\left[\frac{\xi^2}{2\pi}k\omega m|\Psi|^2\right]^{1/2}h^3+\frac{1}{4}\left[\frac{\xi^2}{2\pi}k\omega m|\Psi|^2\right]^{1/2}\left(\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2+\right.$$

$$\left.\left(\beta l_p^2\frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2\right)^{3/2}h^4$$

is the Mexican hat potential playing the role of the Higgs mechanism, $\bar{\psi}_L^i Y_{ij}\phi\psi_R^i$ is the Higgs-fermion interaction, ψ_L^i are left-handed fermion fields, ψ_R^i are right-handed fermion fields, Y_{ij} is the mixed matrix and Ψ is the wave function of each region of the multiverse. This equation expresses the amplitude of transition of the vacuum from one configuration to another configuration, in terms of Feynman's path integrals computed on all the modes in which the fields of the vacuum could evolve. We must point out that the equation of everything (28) allows us to treat processes in a coherent way, both in the quantum gravity regime by considering the universal generalized Compton wavelength (5), and in the peculiar regimes where the generalized Compton wavelength may be approximated with the Schwarzschild radius or with the standard Compton wavelength.

It is interesting to explore how the approach of the 3D DQV based on the equation of everything (28) allows us to face some problems of the Standard Model. As well known, the Standard Model is affected by a hierarchy problem, i.e., in the light of the absence of new-physics signatures at the TeV scale and beyond, the observed Higgs mass appears rather "unnatural" [47], and in current models – such as supersymmetric extensions [48-51], strong dynamics or technicolor models [52, 53] composite Higgs [54-56] and extra dimensions [57-59] – the question of vacuum stability depends on whether the Higgs self-coupling crosses zero or not deep in the ultraviolet and involves a delicate balance of Standard Model parameters, i.e. implies the necessity of a fine-tuning, of choosing very carefully the parameters of new high-scale physics in order to result in the observed low-energy parameters. Instead, we must remark that our approach of the 3D DQV based on the equation of everything (28), in which the elementary constituents are sub-Planckian black holes of the variable energy density defined at the generalized Compton wavelength (5), does not present the hierarchy problem, in the sense that the different scales – such as the value of the Higgs mass – must not be justified by invoking procedures of fine-tuning but rather emerge as a natural consequence of the theory. In the approach based on the equation of everything (28), the different scales naturally emerge

as a consequence of the peculiar Feynman's path integrals present in the computation, and thus of the specific modes in which the fields of the vacuum could evolve.

The equation of everything (28) leads also to directly derive a gravito-electroweak unification scale M_{unif} by starting from the generalized Compton wavelength and therefore as a phenomenon determined by the virtual "sub-Planckian black holes of the variable energy density of the three-dimensional dynamic quantum vacuum", in terms of relation

$$M_{unif} = \frac{1}{2}\left(\frac{3\hbar^2 M_{Pl}^2}{\pi c^2\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qv}E^V}\right)^2 + \left(\beta l_P^2 \frac{\Delta\rho_{qv}E^V}{\hbar\ c}\right)^2\right]}\right)^{1/4} \quad (29).$$

On the basis of the mass unification scale (29), the mass-squared differences of neutrinos determined by fluctuations of the quantum vacuum energy density associated with the virtual "sub-Planckian black holes" of the variable energy density of the 3D DQV, lead to corresponding values of the unification scale for solar and atmospheric neutrinos that turn out to be in good agreement with the results obtained in [60] as well as in other previous theories of electroweak unification [61-63].

On the other hand, in the presence of quantum gravity, namely if the Planck mass is not negligible, we know that in a perturbative approach, when the gauge coupling g of matter inside a $U(1)$ theory tends to zero, a cut-off m_0, which typically corresponds to the mass of the lightest state in an infinite tower of states, appears, namely

$$g \to 0 \implies m_0 \lesssim g M_{Pl} \quad (30).$$

This physics cut-off turns out to be associated to infinite tower of states, such as string or Kaluza-Klein excitations that have to put towards a higher dimension in order to face the issue [64-69]. In the picture of an effective field theory coupled with the gravitational field the presence of a cut-off m_0 turns out to be unfamiliar, in the sense that determines the risk of the gauge interaction becoming weaker than gravity. Now, we must emphasize that, in the approach based on the equation of everything (28), this peculiar problem is not present, in the sense that the cut-off m_0 turns out to be a direct consequence of the theory.

In the model of the 3D DQV based on the equation of everything (28) where the elementary constituents are sub-Planckian black holes with a size of order of the generalized Compton wavelength, we underline that the emergence of gravitation and quantum theory can also be put in an intriguing parallelism with the picture presented by the spectral action principle of Connes' non-commutative geometry, which leads to a pre-quantum, pre-spacetime theory.

Taking account of the results obtained in [70], sub-Planckian black holes can be characterized by a trace dynamic action given by relation

$$S = \hbar \int \frac{d\tau}{t_p} \sum_i Tr \left(\frac{l_p^2}{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE^V}}{\hbar \ c}\right)^2} \right) [\dot{q}_{Bi}^2] \quad (31)$$

where $\dot{q}_{Bi} = D \sqrt{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE^V}}{\hbar c}\right)^2}$, D being the Dirac bosonic operator, l_p is Planck length, τ is the Connes time parameter that represents a specific feature of non-commutative geometry. The spectral action principle can be also invoked by considering the Yang-Mills gauge fields, in the sense that the gauge field $F_{\mu\nu}$ can be seen as a modification of the Dirac operator: $D \rightarrow D + F_{\mu\nu}$, whose eigenvalues are raised to the status of matrices, that are identified with the configuration variables q_{Bi} of the corresponding Dirac operators $\dot{q}_{Bi}$. Matter (fermionic) degrees of freedom q_{Fi} and $\dot{q}_{Fi}$ can be introduced such that the eigenvalues of these matrices are connected to the classical relativistic action of point particles. The dynamic action (31) of the sub-Planckian black holes leads to directly derive classical space-time and general relativity, since the Dirac operators collapse to (distinct) eigenvalues, and the spectral action principle ensures recovery of the Einstein-Hilbert action coupled to Yang-Mills fields and relativistic point particles, with additional corrections. Those degrees of freedom which are not sufficiently entangled continue to obey the underlying trace dynamics but can be described to a good approximation by conventional quantum field theory on a classical spacetime background.

The equation of everything (28) allows us to obtain the different regimes of physics under opportune constraints without the problems which regard the effective field theories approach. In particular, for $\frac{\Delta\rho_{qvE^V}}{c^2} \gg M_{Pl}$ we enter the classical general-relativistic regime, where only the paths corresponding to negligible quantum effects are active for the processes into consideration. In this regime, the amplitude (28) of transition of the vacuum from one configuration to another configuration may be approximated as

$$W_{grav} = \int [Dg][D\psi][D\Psi][D\phi]\, exp\left\{\frac{i}{\hbar}\int\left(\left[1-\frac{\sqrt{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE^V}}{\hbar c}\right)^2}}{r}+\varepsilon\frac{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE^V}}{\hbar c}\right)^2}{4r^2}\right]c^2dt^2-\left[1-\frac{\sqrt{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE^V}}{\hbar c}\right)^2}}{r}+\varepsilon\frac{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE^V}}{\hbar c}\right)^2}{4r^2}\right]^{-1}dr^2-r^2d\Omega^2\right)\sqrt{-g}\left(\frac{c^4}{8\pi G}R-\Lambda\right)\right\} \quad (32)$$

where Λ is the cosmological constant, that here corresponds to peculiar quantum vacuum energy density fluctuations given by relation

$$\rho_{DE} \simeq \frac{35Gc^2}{2\pi\hbar^4 V}\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right) \quad (33)$$

and that in our model of 3D DQV emerges as the fundamental origin of the curvature of space-time characteristic of general relativity [23-26, 28, 29].

Instead, for $\frac{\Delta\rho_{qvE^V}}{c^2} \ll M_{Pl}$ we are in the quantum regime of elementary particles and, in this context, the low energies of the Standard Model correspond to integrals which have a limit, namely an ultra-violet cut-off. In this regard, in particular, the spontaneous symmetry breaking generating the TeV scale of strong interactions occurs when the potential appearing in (28) tend to the effective potential given by (25), thus leading to the expression (27) for the Higgs mass.

However, in the regime of quantum gravity, one must consider all the contributions, at all energies, in these path integrals, on must consider all the modes in which the fields of the vacuum could evolve. Moreover, it must be emphasized that the limit of the theory for $\beta \to 0$ coincides with the results of non-relativistic quantum mechanics; instead, as regards the limit $\varepsilon \to 0$ in the regime $\frac{\Delta\rho_{qvE^V}}{c^2} \gg M_{Pl}$ where the generalized Compton wavelength tends to the standard Schwarzschild radius, the results are in good agreement with general relativity, and in the regime $\Delta\rho_{qvE}V \ll M_{Pl}c^2$ where the generalized Compton wavelength tends to the Compton wavelength, one obtains the regime of usual elementary particles of the Standard Model.

As regards dark matter and dark energy, it must be emphasized that here these are not mysterious entities which are extraneous, bizarre with respect to the formalism of the theory but are instead as a natural consequence/complement in the sense that can be seen as the results of peculiar fluctuations of the quantum vacuum energy density.

Finally, it is interesting to observe that, on the basis of its features and its consequences in the different regimes, the equation of everything (28) fits naturally also as an useful tool for cosmology. In this regard, one can provide the dynamics laws for matter and gravity, by considering a partition function of the form:

$$Z = \int Dq e^{\frac{i}{\hbar}W} \quad (34)$$

where q contains both matter and gravity variables, eventually incorporating dark matter as the results of peculiar fluctuations of the quantum vacuum energy density. Moreover, the laws for the boundary condition of the universe can be defined in terms of double path integrals of the form

$$D[q] = \int Dq' \int Dq e^{\frac{i}{\hbar}(W[q]-W[q'])}\rho(q_b, q_b') \quad (35)$$

where the path integral is doubled to account for the possible mixedness of the boundary condition ρ, which as an analog of the density operator takes as inputs the boundary configurations q_b and q_b' in the double copies.

On the basis of the partition function (34) and taking account of the Quantum Boltzmann Statistics characterizing the micro-universes of the 3D DQV, one finds new keys of explanation of the different features of the parallel universes in a more fundamental picture with respect to superstring theory and eternal inflationary model, in the sense that here the different evolutionary possibilities, such as expansion and contraction, as well as the different values of the cosmological constant of the parallel universes, are directly fixed by (and thus can be seen as the direct consequence of) the different specific behaviours of the elementary energetic fluctuations of the 3D DQV [35]. By considering the following semiclassical approximation of the wave function of the k-universe

$$\Psi_k \approx e^{\pm\frac{i}{\hbar}S_0(a)}Z_k(a,\varphi) \quad (36)$$

with S_0 being the action of the gravitational part alone with no interaction and the positive and negative signs in equation corresponding to the contracting and the expanding branches of Ψ_k, respectively, and by defining

$$\tilde{V}_k(a,\varphi) = V(\varphi) + \frac{\lambda^2(a)}{4\pi^2 a^4} sin^2\left(\frac{\pi k}{N}\right) \quad (37)$$

where $V(\varphi)$ is a potential of the scalar field, one obtains a Schrödinger-type equation for the partition function associated with the k-universe of the form

$$-i\hbar\frac{\partial}{\partial t}Z_k = \frac{1}{2}\left(\frac{1}{a}\frac{\partial^2}{\partial\varphi^2} + a\tilde{V}_k(a,\varphi)\right)Z_k \quad (38)$$

and the following field equation for the scalar field

$$\ddot{\varphi} + 3\frac{\dot{a}}{a}\dot{\varphi} + \frac{d\tilde{V}_k}{d\varphi} = 0 \quad (39)$$

where the dots stands for derivative with respect to the Friedmann time t, i.e. $\dot{\varphi} \equiv \frac{d\varphi}{dt}$. As regards the field equation (39), one can remark that, while the classical behaviour of the scalar field remains unaltered with respect the usual

description because the last term in the potential $\tilde{V}_k$, given by equation (37), has no influence upon equation (39), instead, in the quantum regime, the extra term in the potential induces a modification in the vacuum state that has to be accounted for any vacuum decay process of the universe, resulting in the process of bubble formation and in a suggestive construction of the global structure of the 3D DQV.

As a consequence of the different possible behaviours of the quantum vacuum energy density fluctuations as well as of the Quantum Boltzmann Statistics characterizing the micro-universes of the 3D DQV, the process of bubble formation and the global structure of a single universe is very rich, in the sense that small baby universes are created from quantum fluctuations of the background. At small values of the scale factor (i.e. values close to the Planck scale) the fluctuations of the scalar field and the effects of the interaction among the universes are dominant, so the newborn universes are expected to remain in normal modes with a high value of k. In other words, our universe can be seen as a large parent universe propagating in a plasma of baby universes. In this picture, the process of creation of these newborn bubbles can be considered as an eternal process, which gives origin in a cyclic way to new systems of space and matter in dynamic equilibrium.

Moreover, as regards the issue of cosmological constant, as a consequence of the different possible behaviours of the partition function (34) and therefore of the quantum vacuum energy density fluctuations, universes with different values of the cosmological constant are possible. In this regard, on the basis of the results obtained in [71], if a universe was born in a metastable false vacuum state represented by the local minimum of a canonical scalar field describing the current stage of accelerated expansion of the universe (and which can be identified with the density of the dark energy), one can suppose that successively there was a relaxing to the small values of today, as a consequence of the fact that the value of the cosmological constant is characterized by a diminishing during the post-inflationary phase.

4. The interpretation of time

The standard WDW equation as well as relevant research on quantum gravity imply that time, at a fundamental level, cannot be considered a primary physical reality but represents only an emerging feature, i.e. that the ultimate background of physical processes, at a fundamental level, is timeless. Contemporary theoretical physics leads to the result that time has not a fundamental citizenship in physics, pointing towards the so-called "frozen formalism" which apparently tells that nothing should happen in the universe at the ultimate level, therefore generating the problem regarding how one can extract dynamics from a timeless non-dynamical space. In this regard, the two most famous attempts to address the so-called "problem of time" existing in the

literature are Barbour's "no-time interpretation" and Rovelli's thermal time hypothesis [72-83]. Barbour's approach suggests a Machian resolution to the problem of time by replacing time with a reparametrization invariant action that is integrated over an unphysical evolution parameter, called the ephemeris time, that provides a metric-trans-temporal notion of identity between two subsequent configurations, thus implying a complete emergence of time from the dynamics. Rovelli's "thermal time hypothesis", lies in the idea that the familiar notion of time emerges at the thermodynamic level, when one provides an approximate statistical description of a system with a large number of degrees of freedom, that what we call "time" is the thermal time of the statistical state in which the world happens to be, when described in terms of the macroscopic parameters we have chosen, in summary that time is the expression of our ignorance of the full microstate, a reflex of our incomplete knowledge of the state of the world. According to Rovelli's approach, an idealized time is not defined at the fundamental level (at the Planck scale), namely, in the quantum gravity regime, time should be simply forgotten.

In the theory of everything of the 3D DQV, where the elementary constituents are sub-Planckian black holes of the variable energy density and each regime of physical processes is a consequence of the equation of everything (28), one can go a step ahead with respect to Barbour's and Rovelli's theories and provide a unifying re-reading of these theories, in the sense that time exists only as a mathematical quantity measuring the numerical order of events, that, at the fundamental level, is an emergent feature from the deepest processes occurring at the generalized Compton wavelength. In our approach, we consider the possibility that time measured by clocks can be interpreted as a counter function of the number of states regarding the dynamics of the system under consideration, as a phenomenon that derives from the fundamental variable quantum vacuum energy density. We suggest that time is a mathematical quantity emerging from the frequency modes of the 3D DQV associated with the variations of the quantum vacuum energy density with respect to its ground state defined by the Planck energy density (4) at the fundamental scale represented by the generalized Compton wavelength.

In the geometry of the 3D DQV, the entanglement between the clock and the evolving system whose dynamics is studied generates a numerical order of material changes of this system that corresponds to dissipative features of the vacuum near the Planck scale that can be expressed by an opportune dispersion relation of the form

$$\omega^2 = c^2k^2 - \frac{c^4\hbar^2}{\Delta\rho_{qvE}{}^2V^2}k^4 \quad (40)$$

where the wave number k is assimilated just to the inverse of the generalized Compton wavelength (5), yielding

$$\omega^2 = \frac{c^2}{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2} - \frac{c^4\hbar^2}{\Delta\rho_{qvE}{}^2V^2\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2\right]^2} \quad (41)\ [46].$$

The dissipative features of the 3D DQV described by the dispersion relation (41) imply that the states of the clock are ruled by a non-linear Schrödinger equation, a peculiar version of the Gross–Pitaevskij equation of the form

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m}\nabla^2\psi +$$

$$\frac{a^2}{2\pi}\left(\sqrt{\frac{c^2}{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2} - \frac{c^4\hbar^2}{\Delta\rho_{qvE}{}^2V^2\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2\right]^2}}\right) m|\psi|^2\psi +$$

$$U\psi \quad (42)$$

where m is the mass of each sub-planckian black hole of the physical vacuum, U is the potential energy relating to the single virtual particle and ν is a viscosity coefficient having the dimensions $\frac{length^2}{time}$ which can be expressed as $\nu = \frac{a^2\omega}{2\pi}$ where a is the scattering length between the virtual particles. As a consequence of the dissipative processes of the vacuum described by the Gross-Pitaevskij equation (42), the time marked by the clock has the physical meaning of numerical order of material change associated with the dynamics of the system under study, that can be associated to the inverse of the frequency defined by the dispersion relation (41), namely

$$t = \frac{1}{\sqrt{\frac{c^2}{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2} - \frac{c^4\hbar^2}{\Delta\rho_{qvE}{}^2V^2\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2\right]^2}}} \quad (43)$$

and can be associated to a quantum observable $\hat{\mathcal{T}}$ as follows:

$$\hat{\mathcal{T}}|t\rangle_C = \frac{1}{\sqrt{\frac{c^2}{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2} - \frac{c^4\hbar^2}{\Delta\rho_{qvE}{}^2V^2\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2\right]^2}}}|t\rangle_C \quad (44).$$

The physical meaning of equation (44) is just that the entanglement between the evolving system and the clock generates a counter function labelled by the hands of the clock which is linked with the frequency modes of the 3D DQV, namely with the variations of the quantum vacuum energy density, that are ultimately determined by the dissipative features of the vacuum at the generalized Compton wavelength.

Moreover, taking account the results obtained in [84], we assume that the observable $\hat{H}_C$ and the clock's observable $\hat{\mathcal{T}}$ obey the commutation relation

$$[\hat{\mathcal{T}}, \hat{H}_C] = i\hbar \quad (45)$$

which physically means that $\hat{H}_C$ is the generator of displacements in the position of the clock's hands, i.e. that for an infinitesimal displacement dt it follows that $e^{-i\hat{H}_C dt/\epsilon\hbar}|t\rangle_C = |t+dt\rangle_C$. As a consequence, the state of the evolving system under study Γ when the clock reads t, that is, when the clock is in the state $|t\rangle_C$, is obtained by projecting $|\Psi\rangle$ onto $|t\rangle_C$ and can be denoted as $|\phi(t)\rangle_\Gamma = \langle t|\Psi\rangle$. In this way we obtain:

$$\frac{\partial}{\partial\varphi}|\phi(t)\rangle_\Gamma = \frac{i}{\hbar}\langle t|\hat{H}_C|\Psi\rangle \quad (46)$$

Now, if we refer to Page's and Wooters' conditional probabilities approach (which provides a way to grasp the idea that the passage of time should be identified with correlations inside the system rather than reflecting changes with respect to an external parameter, thus seeming well-suited for application to the problem of time in quantum gravity), the total Hamiltonian of the composite system evolving system + clock can be expressed as

$$\hat{H} = \hat{H}_C \otimes \hat{\mathbb{I}}_\Gamma + \hat{\mathbb{I}}_C \otimes \hat{H}_\Gamma \quad (47)$$

namely one has $\hat{H}_C \otimes \hat{\mathbb{I}}_\Gamma = \hat{H} - \hat{\mathbb{I}}_C \otimes \hat{H}_\Gamma$ and $\hat{H}|\Psi\rangle = 0$. Therefore, after some algebra, one obtains

$$\begin{aligned}\frac{\partial}{\partial t}|\phi(t)\rangle_\Gamma &= \frac{i}{\hbar}\langle t|\hat{H} - \hat{\mathbb{I}}_C \otimes \hat{H}_\Gamma|\Psi\rangle = \\ &= \frac{-i}{\hbar}\langle t|\hat{\mathbb{I}}_C \otimes \hat{H}_\Gamma|\Psi\rangle = \\ &= \frac{-i}{\hbar}\hat{H}_\Gamma\langle t|\Psi\rangle = \\ &\frac{-i}{\hbar}\hat{H}_\Gamma|\phi(t)\rangle_\Gamma \quad (48)\end{aligned}$$

namely

$$i\hbar\frac{d}{dt}|\phi(t)\rangle_\Gamma = \hat{H}_\Gamma|\phi(t)\rangle_\Gamma \quad (49)$$

which is the usual Schrödinger equation for the state of the evolving system. Thus, in the model of the 3D DQV where time is a measure of the numerical order of material change associated with dissipative features of the vacuum at the generalized Compton wavelength, by substituting (43) inside equation (49), the Schrödinger equation (49) reads

$$i\hbar \frac{d}{d\left(\sqrt{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE}V}{\hbar\ c}\right)^2}\ \Delta\rho_{qvE}{}^2V^2\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE}V}{\hbar\ c}\right)^2\right]^2\right)} \frac{1}{c^2 \quad c^4\hbar^2} |\phi(t)\rangle_\Gamma =$$

$$\hat{H}_\Gamma|\phi(t)\rangle_\Gamma \quad (50).$$

Equation (50) can be considered as the "timeless" version of the Schrödinger equation in the 3D DQV.

5. Towards new solutions of the paradoxes of black hole complementarity

In the model of the 3D DQV, the fundamental background of processes is represented by elementary objects of the Planck scale, namely sub-Planckian black holes with a size of order of their Compton wavelength that are generated by the geometrodynamic properties of the variable quantum vacuum energy density ultimately associated to the foam of the virtual particles of the vacuum. In this picture, all black holes have an ultimate quantum nature and each subatomic particle can be seen as something similar to a black hole. On the other hand, in the Elementary Cycles Theory developed in some recent papers [85, 86], which posits a unified formulation of relativistic and quantum physics as well as a fully geometrodynamical formulation of gauge interactions, every isolated elementary constituent of nature is characterized by a persistent intrinsic Compton periodicity $T_C = \frac{h}{Mc^2}$, where M is the mass of the particle. Therefore, on the basis of these results, it seems legitimate to consider the possibility that sub-Planckian black holes of the variable quantum vacuum energy density are characterized by a cyclic intrinsic Compton periodicity, which thus emerges as the fundamental connecting element between the particle aspect and the black hole aspect of these elementary objects. On the basis of the treatment made in chapter 4, in our approach, we can consider that the ultimate intrinsic Compton periodicity of the sub-planckian black holes of the variable quantum vacuum energy density is ultimately associated with the dissipative features of the vacuum at the generalized Compton wavelength and expressed by equation (43).

As regards the physics of black holes, quantum mechanics and general relativity have generated relevant conflicts that yielded the two famous paradoxes represented by the black hole information paradox and firewall paradox. On the basis of the Hawking idea of evaporation of semi-classical black holes, the problem is originated of whether the quantum information contained in the matter that collapsed to form the black hole is destroyed and lost forever. The study of the evolution of gravitational collapse through classical models turns out to be deeply unsatisfactory. If at the beginning of the collapse one deals with the formation of a naked singularity [87] then, by considering a semi-classical treatment of the radiation (assuming the validity of effective field theory), from the singularity it is not clear what the final fate of such a collapse is: either the collapsing star will dissipate entirely or a remnant will attempt to generate a covered singularity. Instead, if at the beginning of the collapse one has a covered singularity and the formation of an event horizon then, as Hawking pointed out [88, 89], the semi-classical theory would yield thermal radiation from the point of view of the observer who remains outside the black hole provided that the freely falling observer detects nothing unusual

while crossing the horizon. The semi-classical analysis would seem to suggest that information is lost if the black hole evaporates completely.

On the other hand, the Hawking radiation carrying information was reconciled with the equivalence principle in general relativity by means of the concept of black hole complementarity [90, 91], according to which a freely falling observer crossing the horizon would detect nothing out of the ordinary, while a fixed observer would detect a significant feature known as the "stretched horizon." According to the Susskind and Thorlacius model, an observer who falls into a black hole after extracting information from the Hawking radiation would not be able to detect the same information inside the black hole before reaching the spacelike singularity, considering that information absorbed by the black hole can be reconstructed after the Page time [92, 93]. Instead, Hayden and Preskill [94] suggested that, if information is thrown into a black hole after the Page time, it can still be recovered after the so-called scrambling time, which is typically less than the Page time, thus evading the duplication of information.

Now, in this chapter, we want to show how the model of the 3D DQV based on the equation of everything (28) and thus on the sub-planckian black holes of the variable quantum vacuum energy density defined by the intrinsic periodicity (43) associated with the dissipative features of the vacuum at the generalized Compton wavelength, suggests new perspectives of solution to the paradoxes connected with the physics of black holes. In particular, here the key point is that the wave function of each micro-universe associated with peculiar path integrals that appear in the equation of everything (28) and the consequent interpretation of time as a sequential numerical order of material change emerging from the frequency modes of the 3D DQV associated with the variations of the quantum vacuum energy density, allow us to avoid the paradoxes connected with the simultaneous consideration of the three fundamental assumptions underlying black hole complementarity invoked by Almheiri et. al. (AMPS) [95], i.e.:

1. Hawking radiation is pure;
2. Effective field theory is valid outside a stretched horizon;
3. infalling observers encounter nothing unusual as they cross the horizon.

In the light of Hawking's results [96], the action of the horizon as a "firewall" proposed by Almheiri violates the equivalence principle and breaks the CPT invariance of quantum gravity. In this way, Hawking suggested the idea that no real event horizon is formed and that somehow collapse does not continue beyond an apparent horizon. Recently, Vaz developed the Hawking idea of the apparent horizon as final result of the gravitational collapse by considering a quantum gravitational model of dust collapse [97], which implies that a continued collapse towards a singularity can only be obtained if one combines two independent and entire solutions of the Wheeler-DeWitt equation. By developing a quantum approach to the spherical collapse of inhomogeneous dust in AdS black holes of dimension $d=n+2$, that is described

by the Lemaitre-Bondi-Tolman family of metrics, Vaz found two independent solutions of Wheeler-DeWitt equation in terms of shell wave functions:

$$\psi_i = \psi_i^{(1)} + A_i \psi_i^{(2)} \quad (51)$$

where $\psi_i^{(1)}$ represents dust shells condensing to the apparent horizon on both sides of it and $\psi_i^{(2)}$ represents dust shells that move away from the apparent horizon on either side of it where the exterior, outgoing wave is suppressed by the Boltzmann factor at the Hawking temperature for the shell, and in this picture the AMPS firewall paradox is avoided if A_i tends to zero, thus making $\psi_i^{(1)}$ the complete description of the quantum collapse till the apparent horizon.

Now, in our model of 3D DQV ruled by the generalized Compton wavelength and by the equation of everything (28), we are able to provide an alternative solution to the paradoxes connected to the black hole complementarity. In our approach, the specific active path integrals in the equation of everything (28) as well as the idea of time, and thus of the intrinsic periodicity of the sub-Planckian black holes of the variable energy density of the 3D DQV, acting as numerical order of material change expressed by equation (43), imply that the three fundamental assumptions underlying black hole complementarity are not necessarily incompatible: the final result of the gravitational collapse is an extremely compact quantum object that is held up not by any degeneracy pressure but by quantum gravity processes in the same way that ordinary atoms are sustained by quantum mechanics. More precisely, we deal here with the formation of compact quantum objects as a consequence of the dissipative features of the vacuum at the generalized Compton wavelength.

Here, the standard Wheeler-DeWitt equation is replaced by the equation of background (8) whose general solutions are the wave functions (9) and these wave functions correspond to definite active path integrals in the equation of everything (28). As a consequence, the gravitational collapse characterizing the physics of black holes can be described by means of opportune active path integrals deriving from the equation of everything (28). The active path integrals describing the gravitational collapse of black holes correspond to a lattice of collapsing dust shells, namely micro-universes that, by indicating $\sqrt{\frac{c^2}{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE}V}{\hbar\ c}\right)^2}-\frac{c^4\hbar^2}{\Delta\rho_{qvE}{}^2V^2\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE}V}{\hbar\ c}\right)^2\right]^2}} = \xi$, are defined by the following wave functions

$$\Psi_{\pm} = \begin{cases} \frac{V\Delta\rho_{qvE}}{\sqrt{2\pi}\hbar l_p{}^2} e^{\hbar\xi d} \int d\rho \left(e^{\pm i\frac{V\Delta\rho_{qvE}}{\hbar l_p{}^2}\varphi} \Psi_{\Delta\rho_{qvE}}(a)\hat{b}_{\Delta\rho_{qvE}} + e^{\mp i\frac{V\Delta\rho_{qvE}}{\hbar l_p{}^2}\varphi} \Psi^*_{\Delta\rho_{qvE}}(a)\hat{b}^{\dagger}_{\Delta\rho_{qvE}} \right) \\ \frac{V\Delta\rho_{qvE}}{\sqrt{2\pi}\hbar l_p{}^2} e^{\hbar\xi\left(d-\frac{\pi}{\hbar g\hbar}\right)} \int d\rho \left(e^{\pm i\frac{V\Delta\rho_{qvE}}{\hbar l_p{}^2}\varphi} \Psi_{\Delta\rho_{qvE}}(a)\hat{b}_{\Delta\rho_{qvE}} + e^{\mp i\frac{V\Delta\rho_{qvE}}{\hbar l_p{}^2}\varphi} \Psi^*_{\Delta\rho_{qvE}}(a)\hat{b}^{\dagger}_{\Delta\rho_{qvE}} \right) \end{cases} \quad (52)$$

where g_h is the surface gravity at the apparent horizon and d is an opportune parameter $d = d(\Delta\rho_{qvE})$ that depends of the variable energy density corresponding to the formation of the shells. The physical meaning of equation (52) is simple: the micro-universes described by equation (52) represent dust shells condensing to the apparent horizon. These shells have bound states of energy that can be determined by considering the following potential of interaction between the fluctuations of the quantum vacuum energy density and the quantum compact object

$$V(r) = -G\frac{\Delta\rho_{qvE}Vm}{2Rc^2} \quad (53)$$

where m is the mass of the dark compact object and R is the radius of the spherical massive shell. In analogy to what happens in the hydrogen atom, on the basis of the potential of interaction (53), one obtains the following bound states of energy of the dust shells:

$$E_n = -\frac{1}{n^2}\frac{G^2m^3\Delta\rho_{qvE}^2V^2}{2\hbar^2c^4} \quad (54).$$

The energy spectrum (54) leads to the following mass spectrum of the shells:

$$m_n = \frac{2}{n^2}\frac{G^2\Delta\rho_{qvE}^4V^4}{\hbar^2c^4} \quad (55).$$

The position of the physical quantum shells cannot be exactly localized but can be defined through an average radius expressed by relation

$$\langle R_n\rangle = \frac{3}{2}\frac{m_nc^2}{\Delta\rho_{qvE}V}\sqrt{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2 + \left(\beta l_p^2\frac{\Delta\rho_{qvE}V}{\hbar c}\right)^2} \quad (56).$$

Now, by following [98], we can compute the minimum value of the shells in Planck units as the difference between the volume of the sphere having radius $\langle R_n\rangle + \frac{1}{2}l_p$ and the volume of the sphere having radius $\langle R_n\rangle - \frac{1}{2}l_p$:

$$V_{min} = \frac{4}{3}\pi\left[\left(\langle R_n\rangle + \frac{1}{2}l_p\right)^3 - \left(\langle R_n\rangle - \frac{1}{2}l_p\right)^3\right] = \frac{4}{3}\pi\left[3\langle R_n\rangle^2l_p + \frac{1}{4}l_p^3\right] = 4\pi\langle R_n\rangle^2l_p + \frac{\pi}{3}l_p^3 \quad (57).$$

Therefore, taking account of the mass spectrum (55), one finds the following expression for the maximum value of the density of the shells:

$$\sigma_{max} = \frac{2G^2\Delta\rho_{qvE}^4V^4}{n^2\hbar^2c^4\left(4\pi\langle R_n\rangle^2l_p + \frac{\pi}{3}l_p^3\right)} \quad (58)$$

which shows that the maximum density is decreasing with increasing of the quantum number n of the shells, as one intuitively expects.

The physical meaning of the approach based on equations (52)-(58) is that, in the light of the active path integrals appearing in the equation of everything (28) and the consequent role of time as numerical order of material change emerging from the dissipative features of the 3D DQV at the generalized Compton wavelength, each shell will condense to the apparent horizon and will not undergo further collapse, namely there is no tunneling into the exterior and no firewall. On the basis of the wave functions (52) of the collapsing dust shells,

the AMPS paradox is directly avoided without further assumptions, in the sense that the paradox connected to the black hole complementarity are not present in the wave functions (52). If the gravitational collapse is described by the wave function (52), the dissipative features of the 3D DQV at the generalized Compton wavelength determine the absence of thermal radiation outside and the presence of a thermal Unruh radiation inside the apparent horizon, therefore implying the non existence of a continuous collapse to a central singularity, in the sense that the collapse would terminate at the apparent horizon, and this result turns out to be in good agreement with the original Hawking proposal [96, 97].

Moreover, as a result of the formation of the apparent horizon determined by the dissipative features of the 3D DQV at the generalized Compton wavelength, the interior spacetime of the black hole emerges as an effective description of a given exterior time in the sense that the apparent horizon indeed produces an entanglement between the information inside the black hole and the information outside the black hole. In this way, the entanglement produced by the apparent horizon leads to the generation of an effective time evolution foliating the interior region that is different from the more fundamental, exterior time evolution, which foliates only the spacetime region outside the horizon. In other words, we can say that the action of the specific path integrals of equation (28) and the consequent of time as numerical order of material change generated by the dissipative features of the vacuum at the generalized Compton wavelength leads to the result that any information in the effective interior is encoded in the modes outside the horizon. As a consequence, the evolution of the system after the formation of the apparent horizon can be seen as a perspective effect owed to the dissipative features of the vacuum at the generalized Compton wavelength, that indeed generate the connection between the information contained in the effective interior of the black hole and the modes outside the apparent horizon. The surface of the apparent horizon, in the light of the dissipative features of the 3D DQV at the generalized Compton wavelength, acts as a sort of entanglement island, implying that some information about the interior might be detectable on the surface of the apparent horizon, where pairs of virtual particles jump in and out of existence in such a way that, if one of these quantum entangled pairs is split up by the apparent horizon and will be held in the black hole, then the other will fly out as Hawking radiation. The entanglement effect determined into the surface of the apparent horizon associated with the processes of collapsing dust shells by the dissipative features of the vacuum at the generalized Compton wavelength, can therefore provide a new key of reading, at a deeper level, of the results recently obtained by Bousso and Penington, regarding the existence of entanglement surfaces that exist slightly outside the event horizon and that, for a supermassive black hole, could extend as much as an atom's length [99].

On the other hand, the entanglement effect between interior and exterior emerging from the peculiar path integrals of the equation of everything (28) that lead to the lattice of collapsing dust shells, can be also seen as a direct consequence of the action of the quantum Boltzmann statistics satisfied by the creation and annihilation operators appearing in the wave functions (52) that describe the gravitational collapse of the quantum shells till the apparent horizon. Moreover, by following [100], the entanglement effect between interior information and exterior modes can be associated with the action of the area of the apparent horizon – expressed in terms of opportune condensates of elementary grains having the size of the generalized Compton wavelength – as a coarse-grained entropy:

$$S^{coarse} = \frac{k\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE^V}}{\hbar c}\right)^2\right]}{4G\hbar} \quad (59)$$

where k is an opportune parameter characterizing the number of grains constituting the surface of the apparent horizon. On the basis of equation (59), the area of the apparent horizon can be considered as the maximum boundary entropy that is compatible with the geometry of its exterior, making a holographic connection between interior information and exterior information in terms of a set of independent qubits, one per elementary area corresponding to a generalized Compton wavelength.

Because of the origin of time as numerical order of material change, and thus of the intrinsic periodicity of the sub-Planckian black holes of the variable energy density of the 3D DQV, from the dissipative features of the 3D DQV at the generalized Compton wavelength and the consequent processes of entanglement between interior information and exterior information of the apparent horizon, physics characterizing the black hole interior can be associated to a perspective effect, i.e. an effective theory regarding a spacetime region K_{t_x} containing a portion of the

zone $k\sqrt{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE^V}}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE^V}}{\hbar\ c}\right)^2} < r < r_Z$ where r_Z is the location of the outer edge of the near black-hole region inside the barrier of the effective gravitational potential generated by the black hole, only up to a time

$$T_{t_x} = t_x + 2\frac{r_+}{c}\ln\frac{r_+\xi}{c} \quad (60)$$

where r_+ is the radius of the apparent horizon of the black hole corresponding to the time t_x. This means that, as regards the interior of the black hole, the physics characterizing the interior portion of K_{t_x} should be viewed as occurring in the time frame

$$t_x < t < T_{t_x} \quad (61).$$

The action of the dissipative features of the 3D DQV at the generalized Compton wavelength, that generate the entanglement between interior

information and exterior information of the black hole, rearranges correlations between the degrees of freedom associated with the time slices in (61) in such a way that they can be interpreted as originating from the infalling time evolution generated by an opportune operator without involving a further exterior time evolution. Moreover, we must emphasize that, in the exterior description, by virtue of the perspective effect determined by the dissipative features of the vacuum, ultimately derived from the entanglement interaction between interior and exterior produced by the apparent horizon, the physics which an interior observer experiences at a given spacetime point x must have already happened before the evaporation of the black hole. Thus, whatever the detector does to the final state Hawking radiation, it cannot affect their experience at x nor events occurring in its future.

By following [101], one can also extend the analysis to the case in which one deals with operators before the black hole is fully evaporated, by considering an infalling observer who reaches the apparent horizon in correspondence to some exterior time t_f. While the experience of the observer shortly after crossing the horizon is described by the effective theory of the interior corresponding to t_f, this does not necessarily mean that all the experiences of this observer are described in this effective theory. For example, the observer may accelerate toward the horizon right after they cross it, eventually leaving the region K_{t_f} generating the entanglement perspective effect which connects the information contained in the effective interior of the black hole and the modes outside the apparent horizon. In this case, the minimum numerical order needed by outside observers in order to extract quantum information from the black hole after an infalling observer has been absorbed into the apparent horizon is given by relation

$$t_{scr} = 4\frac{r_+}{c}\ln\frac{r_+\xi}{c} \quad (62)$$

that can be defined as the "scrambling time" of black holes in the model of the sub-Planckian black holes of the variable energy density of the 3D DQV. So, the experience of the infallen observer should be viewed as fully completed by the time

$$t_{comp} = t_f + t_{scr} \quad (63)$$

in the exterior description. The experience of the infalling observer can be associated to the time (63) in the sense that the late trajectory of the observer is fully contained in the effective theory associated with the time

$$t_f' = t_f + 2\frac{r_+}{c}\ln\frac{r_+\xi}{c} \quad (64)$$

even in the worst case that the observer starts moving outward at the speed of light crossing the mathematical apparent horizon. On the basis of the argument that led to equation (61), any experience of the observer in the interior must have be associated to the time

$$T_{t_f'} = t_f + t_{scr} \quad (65).$$

On the other hand, if one uses the general argument provided in [102], one can say that the information about an object that fell into the black hole at t_f emerges in Hawking radiation only after $t_f + t_{scr}$ as a consequence of the perspective effect, and therefore of the entanglement between interior and exterior determined by the dissipative features of the vacuum. Therefore, as a consequence of the action of the scrambling time intended as a numerical order of material change associated with the dissipative features of the vacuum at the generalized Compton wavelength, it follows that any operation on Hawking radiation — even if it is performed before the time of evaporation, and no matter how complex it is — cannot affect the experience of an observer in the interior. This conclusion is valid even if we extract radiation from the Hawking cloud in the zone around the black hole [103, 104].

The treatment made in this chapter leads to the result that the black hole's singularity cannot be considered a primary physical reality [105]. From an exterior perspective, an object does not immediately thermalize when it encounters the stretched apparent horizon. Instead, it manifests as a distinct excitation on the horizon for some duration in the form of a perspective effect generated by the dissipative features of the vacuum at the generalized Compton wavelength. Over the scrambling time, t_{scr}, this excitation will gradually dissipate into the horizon. During this period, the dissipation process has an effective description as an object traversing the near-empty interior of the black hole. After the object's excitation is fully integrated into the apparent horizon, the perspective effect associated with the dissipation processes of the vacuum and the corresponding effective theory that describes the connection between the information contained in the effective interior of the black hole and the modes outside the apparent horizon, does not hold anymore.

Because of the features of the black hole evolution which involve collapsing dust shells which terminate at the apparent horizon as a consequence of the dissipative features of the 3D DQV at the generalized Compton wavelength, an exterior observer cannot retrieve information about an object which crossed the apparent horizon at an earlier time. Thus, in this picture, there is no problem of cloning if the observer jumps into the horizon. In other words, an observer cannot obtain a second copy of the information and the no-cloning theorem is naturally saved as a consequence of the entanglement action of the apparent horizon between the information contained in the effective interior of the black hole and the modes outside the apparent horizon. In particular, by substituting the scrambling time (61) inside the generalized energy-time uncertainty relation

$$\Delta E \Delta t \geq \frac{\hbar}{2}\left(1 + \beta l_p^2 \frac{\Delta \rho_{qvE}^2 V^2}{\hbar^2 c^2}\right) \quad (66),$$

the duplication of quantum information is avoided (and therefore the information cloning is evaded) if one deals with a scale of energy

$$\Delta E \gg \frac{\hbar c}{8r_+ \ln\frac{r_+\xi}{c}}\left(1+\beta l_p^2 \frac{\Delta\rho_{qvE}^2 V^2}{\hbar^2 c^2}\right) \quad (67).$$

We underline here that the result (66) turns out to be compatible with the treatment in [106], if the mass of the black hole satisfies relation

$$Mc = \frac{\hbar}{8r_+ \ln\frac{r_+\xi}{c}}\left(1+\beta l_p^2 \frac{\Delta\rho_{qvE}^2 V^2}{\hbar^2 c^2}\right) \quad (68).$$

In summary, on the basis of equations (67) and (68), we can conclude that the cloning of information is avoided if one deals with a scale of energy that exceeds the quantity $\frac{\hbar c}{8r_+ \ln\frac{r_+\xi}{c}}\left(1+\beta l_p^2 \frac{\Delta\rho_{qvE}^2 V^2}{\hbar^2 c^2}\right)$ and this occurs when the scrambling time (62) is sufficiently large.

In the Susskind and Maldacena ER=EPR approach, the black hole could connect to all the particles of Hawking radiation through a virtual wormhole, and each Hawking radiation particle would act as the entrance to the wormhole as well as a large space-time region like the inside of a black hole can be composed of a large number of entangled wormholes, implying that the information can get out of the black hole without causing contradiction of quantum entanglement or opening the firewall [107]. Now, in our approach of 3D DQV ruled by the equation of everything (28), where there is no firewall and no primary singularity, one can go beyond the Susskind-Maldacena ER=EPR approach. The intrinsic periodicity of the sub-Planckian black holes of the variable energy density of the 3D DQV acting as numerical order of material change in the form of equation (43) and generated by the dissipative features of the vacuum at the generalized Compton wavelength, implies that the interior of a black hole emerges as a way of reorganizing the degrees of freedom participating in the transient phenomenon of an infalling object dissipating into the apparent horizon. As a consequence of the entanglement action of the surface of the apparent horizon in generating the connection between the information contained in the effective interior of the black hole and the modes outside the apparent horizon, no operation performed on Hawking radiation, however complex, can affect the experience of an infalling observer. Rather, reconstructions of interior operators from Hawking radiation should be interpreted as acting on a historical record of the effective interior, by means of its connection to the information outside the apparent horizon. While in the effective global picture the interior operators appear spacelike separated from the exterior operators used for reconstruction, in the fundamental picture of the 3D DQV ruled by the generalized Compton wavelength and by the equation of everything (28), they are timelike related as a consequence of the dissipative features of the modes of the vacuum, that give origin to the perspective entanglement effect generating the connection between the information contained in the effective interior of the black hole and the modes outside the apparent horizon.

Finally, we emphasize that, in these processes of collapsing dust shells towards an apparent horizon, the transition to extremely compact quantum objects, ruled by quantum gravity, can be considered as the creation of baby-universes which will separate from a plasma of large parent universes. In this regard, by assuming that the end of the black hole evaporation is associated to a mild singularity, the black hole interior can be seen as a closed universe that is separated from the ambient space describing the exterior of the black hole, compatibly with the interpretation provided in [108], according to which, after the black hole evaporates, its microscopic information is encoded in the states of the baby universes arising in the system of replicated black holes associated with the plasma of large parent universes.

6. Conclusions

In the light of the treatment developed in this paper, the model based on the equation of everything of the 3D DQV (28), that provides the transition amplitude of the vacuum from one configuration to another in terms of Feynman path integrals calculated over all the ways in which the fields of the vacuum could evolve, has the merit to achieve significant results:

- it allows the processes to be treated coherently, both in the regime of quantum gravity (in which the universal generalized Compton relation (5) is considered), and in the particular regimes in which (5) can be approximated by the Schwarzschild radius or by the standard Compton wavelength;
- it avoids the hierarchy problem of the Standard Model, in the sense that the different scales do not require fine-tuning but emerge as a general consequence of the theory, as a consequence of the specific active path integrals;
- it allows us to obtain the different regimes of physics avoiding the problems that are present in effective field theories, under appropriate conditions on the variable quantum vacuum energy density;
- it naturally adapts to cosmology, providing new explanation keys for the different features of parallel universes in a more fundamental framework with respect to superstring theory and eternal inflation, in the sense that the different evolutionary possibilities are fixed by the different specific behaviors of elementary energy fluctuations of the quantum vacuum (in the picture of a the semiclassical approximation (35) of the wave function of the generic universe dependent on the partition function (34) which itself ultimately depends on the transition amplitude of the vacuum, i.e. on the equation of everything (28));

- it allows time to be obtained as the numerical order of the movement associated with the dissipative features of the vacuum generated the variable energy density of the vacuum (on the basis of equation (43)).

On the other hand, the approach of time as a mathematical parameter measuring the numerical order of material change that ultimately originates from the dissipative features of the vacuum at the generalized Compton wavelength, leads to important perspectives as regards the issue of the complementarity of the black holes. The three fundamental assumptions invoked by Almheiri et. al. in [95], i.e.:

1. Hawking radiation is pure;
2. Effective field theory is valid outside a stretched horizon;
3. infalling observers encounter nothing unusual as they cross the horizon

are not necessarily incompatible, in the sense that in our model one deals with a lattice of collapsing dust shells, namely micro-universes and each of these shells will condense to an apparent horizon and will not undergo further collapse, and this implies that, in the physics of the black holes predicted by our theory of everything, the ideas of tunnelling into the exterior and firewall can be naturally avoided. The dissipative features of the 3D DQV at the generalized Compton wavelength determine an entanglement action of the surface of the apparent horizon that generates a connection between the information contained in the effective interior of the black hole and the modes outside the apparent horizon, leading to the result that any information in the effective interior is encoded in the modes outside the horizon. In this picture, the evolution of the system after the formation of the apparent horizon can be seen as a perspective entanglement effect owed to the dissipative features of the vacuum at the generalized Compton wavelength, where correlations between the degrees of freedom associated with the time interval (61) are rearranged in such a way that they can be interpreted as originating from the infalling time evolution generated by an opportune operator without involving a further exterior time evolution. The evolution of the collapsing dust shells which terminate at the apparent horizon as a consequence of the dissipative features of the 3D DQV at the generalized Compton wavelength, implies that an exterior observer cannot retrieve information about an object which crossed the apparent horizon at an earlier time and, therefore, here the no-cloning theorem is naturally saved.

The next issue to be addressed regards the search for a relation between the equation of everything (28) of the 3D DQV model and experience. In this regard, despite the difficulties regarding the selection of the active stories in the path integration, that clarify the relation of the equation of everything (28) with experience, by following [60, 109], one can define the relative probability for some experience in the form of conditional probabilities $p(e_i/c)$ where c is a physical condition (that could be the transient physical configuration of a human

being) that enables a set of possible experiences $\{e_i\}_i$, so being relevant for what the observer experiences next, that is expressed by relation

$$p(e_i/c) = N \int_{e_i,c} Dq' \int_{e_i,c} Dq e^{\frac{i}{\hbar}(W[q]-W[q'])} \rho(q_b, q_b') \quad (69).$$

In equation (69) $W[q]$ and $W[q']$ are the amplitudes of transition of the vacuum from one configuration to another configuration – given in general by equation (28) – corresponding to two different matter and gravity variables, the subscripts e_i, c indicate that, in both branches of the double path integral, we select physical configurations compatible with the condition c and the experience e_i while the normalization constant N is fixed by requiring $\sum_i p(e_i/c) = 1$. On the basis of the conditional probabilities (69), the perspective is opened that, if one being has a given experience e_i with condition c, everywhere else one deals with matter and gravity path integral configurations that coexist in superposition and considerations of other beings' experiences are motivated by the content of the experiences of the first being, in other words no consideration of other beings is required to account for the experiences of one being. Nonetheless, despite the germinal aspects as regards the relation between this theory and experience, the considerations made in this paper can be considered an interesting starting-point towards a new way to formulate an equation of everything in the picture of path integrals, in the light of the relevant unifying results regarding different regimes.

References

1. A. Ashtekar and E. Bianchi, "A short review of loop quantum gravity", arXiv:2104.04394v1 (2021).
2. S. Carlip et al., "Quantum Gravity: A Brief History of Ideas and Some Prospects", *International Journal of Modern Physics D* **24**, 11, 1530028 (2015).
3. F. Close, *Teorie del tutto*, Boringhieri, Torino (2018).
4. B. Greene, *The Elegant Universe: Superstrings, Hidden Dimensions, and the Quest for the Ultimate Theory*, Vintage Books, New York (1999).
5. C. Rovelli, *Quantum gravity*, Cambridge University Press, Cambridge (2004).
6. M. Kaku, *Introduction to Superstrings and M-Theory*, Springer, Berlin (1999).
7. M. Kaku, *L'equazione divina. La ricerca di una teoria del tutto*, Rizzoli, Milano (2021).
8. T. Konopka, F. Markopolou and S. Severini, "Quantum graphity: a model of emergent locality", *Physical Review* **77**, 104029 (2008).

9. D. Oriti (ed.), *Approaches to Quantum Gravity*, Cambridge University Press, Cambridge, UK (2009).
10. D. Oriti, "L'universo emergente della gravità quantistica", *Ithaca: Viaggio nella Scienza* **MCXIV**, 2067 (2018).
11. J.F. Donoghue, in *Effective Field Theories of the Standard Model*, ed. by U.-G. Meissner, World Scientific, Singapore (1992), p. 3.
12. J.F. Donoghue, "General Relativity As An Effective Field Theory: The Leading Quantum Corrections", *Physical Review D* **50**, 3874 (1994).
13. J. F. Donoghue, "Introduction to the effective field theory description of gravity", arXiv:gr-qc/9512024 [gr-qc] (1995).
14. J. F. Donoghue, "The effective field theory treatment of quantum gravity", *AIP Conference Proceedings* **1483**, 1, 73-94 (2012).
15. C. P. Burgess, "Introduction to Effective Field Theory", Cambridge University Press, Cambridge (2020).
16. S. Weinberg, "Effective field theory, past and future," *International Journal of Modern Physics A* **31**, 06, 1630007 (2016).
17. S. Weinberg, "On the Development of Effective Field Theory", *European Physical Journal H* **46**, 1, 6 (2021).
18. A.A. Petrov and A.E. Blechman, *Effective Field Theories*, World Scientific, Singapore (2016).
19. U.G. Meißner and A. Rusetsky, *Effective Field Theories*, Cambridge University Press, Cambridge (2022).
20. S.M. Carroll, *The big picture: on the origins of life, meaning, and the universe itself*, Penguin (2017).
21. S.M. Carroll, "The quantum field theory on which the everyday world supervenes", arXiv:2101.07884v1 [physics.hist-ph] (2021).
22. S. Rivat, "Effective theories and infinite idealizations: a challenge for scientific realism", *Synthese* **198**, 12107-12136 (2021).
23. D. Fiscaletti, *The timeless approach. Frontier perspectives in 21st century physics*, World Scientific, Singapore (2015).
24. D. Fiscaletti, "About dark energy and dark matter in a three-dimensional quantum vacuum model", *Foundations of Physics* **46**, 10, 1307-1340 (2016).
25. D. Fiscaletti and A. Sorli, "About a three-dimensional quantum vacuum as the ultimate origin of gravity, electromagnetic field, dark energy ... and quantum behaviour", *Ukrainian Journal of Physics* **61**, 5, 413-431 (2016).
26. D. Fiscaletti and A. Sorli, "Dynamic quantum vacuum and relativity", *Annales UMCS Sectio AAA: Physica* **LXXI**, pp. 11-52 (2016).
27. D. Fiscaletti and A. Sorli, "About electroweak symmetry breaking, electroweak vacuum and dark matter in a new suggested proposal of completion of the Standard Model in terms of energy fluctuations of a

timeless three-dimensional quantum vacuum", *Quantum Physics Letters* **5**, 3, 55-69 (2016).

28. D. Fiscaletti and A. Sorli, "Quantum vacuum energy density and unifying perspectives between gravity and quantum behaviour of matter", *Annales de la Fondation Louis de Broglie* **42**, 2, 251-297 (2017).
29. D. Fiscaletti and A. Sorli, "Quantum relativity: variable energy density of quantum vacuum as the origin of mass, gravity and the quantum behaviour", *Ukrainian Journal of Physics* **63**, 7, 623-644 (2018).
30. D. Fiscaletti, "About dark matter as an emerging entity from elementary energy density fluctuations of a three-dimensional quantum vacuum", *Journal of Theoretical and Applied Physics* **14**, 203-222 (2020).
31. D. Fiscaletti and A. Sorli, "Generalized uncertainty relations, particles, black holes, and Casimir effect in the three-dimensional quantum vacuum", *Theoretical and Mathematical Physics* **214**, 1, 132-151 (2023).
32. D. Fiscaletti, "Generalized uncertainty relations, generalized Compton wavelength and particles in a quantum foam described by a variable energy density", *Hadronic Journal* **47**, 183-206 (2024).
33. D. Amati, M. Ciafaloni, and G. Veneziano, "Superstring collisions at Planckian energies", *Physics Letters B* **197**, 81-88 (1987).
34. D. Amati, M. Ciafaloni, and G. Veneziano, "Can spacetime be probed below the string size", *Physics Letters B* **216,** 41-47 (1989).
35. M. Maggiore, "A generalized uncertainty principle in quantum gravity", *Physics Letters B* **304**, 65-69 (1993).
36. F. Scardigli, "Generalized uncertainty principle in quantum gravity from micro-black hole gedanken experiment", *Physics Letters B* **452**, 39-44 (1999).
37. S. Capozziello, G. Lambiase, and G. Scarpetta, "Generalized uncertainty principle from quantum geometry", *International Journal of Theoretical Physics* **39**, 15-22 (2000).
38. F. Scardigli and R. Casadio, *Classical Quantum Gravity* **20**, 3915 (2003).
39. R. Casadio, R. Garattini, F. Scardigli, *Physics Letters B* **679**, 156 (2009).
40. R. Casadio, F. Scardigli, *European Physical Journal C* **74**, 2685 (2014).
41. G.G. Luciano and L. Petruziello, "GUP parameter from maximal acceleration", *European Physical Journal C* **79**, 283 (2019).
42. G. Lambiase and F. Scardigli, "Generalized uncertainty principle and asymptotically safe gravity", *Physical Review D* **105**, 124054 (2022).
43. F. Scardigli and R. Casadio, "Gravitational tests of the generalized uncertainty principle", *European Physical Journal C* **75**, 425 (2015).

44. D. Fiscaletti, "Implications of generalized Compton wavelength in the effects of general relativity", *Theoretical and Mathematical Physics*, accepted for publication, 2024.
45. D. Fiscaletti, "About the multiverse in the three-dimensional quantum vacuum with variable energy density", *Bulgarian Journal of Physics* **51**, 2, 185-210 (2024).
46. D. Fiscaletti and A. Sorli, "Perspectives about scales and hierarchies in a three-dimensional quantum vacuum ruled by generalized uncertainty relations", *Bulgarian Journal of Physics* **50**, 4, 412-438 (2023).
47. G.F. Giudice, "Naturalness after LHC8", *PoS* **EPS-HEP2013**, 163 (2013).
48. J. Wess and B. Zumino, "A Lagrangian Model Invariant Under Supergauge Transformations", *Physics Letters B* **49**, 52 (1974).
49. J. Iliopoulos and B. Zumino, "Broken Supergauge Symmetry and Renormalization", *Nuclear Physics B* **76**, 310 (1974).
50. R.K. Kaul, "Gauge Hierarchy in a Supersymmetric Model", *Physics Letters B* **109**, 19-24 (1982).
51. S.P. Martin, "A Supersymmetry primer", *Adv. Ser. Direct. High Energy Phys.* **21**, 1-153 (2010).
52. K. Lane, "Two lectures on Technicolor", arXiv:hep-ph/0202255 (2002).
53. C.T. Hill and E.H. Simmons, "Strong dynamics and electroweak symmetry breaking", *Physics Reports* **381**, 235- 402 (2003).
54. D.B. Kaplan, H. Georgi and S. Dimopoulos, "Composite Higgs Scalars", *Physics Letters B* **136**, 187-190 (1984).
55. M.J. Dugan, H. Georgi and D.B. Kaplan, "Anatomy of a Composite Higgs Model", *Nuclear Physics B* **254**, 299-326 (1985).
56. B. Bellazzini, C. Csaki and J. Serra, "Composite Higgses", *European Physical Journal C* **74**, 5, 2766 (2014).
57. N. Arkani-Hamed, S. Dimopoulos and G. Dvali, "Phenomenology, astrophysics and cosmology of theories with submillimeter dimensions and TeV scale quantum gravity", *Physical Review D* **59**, 086004 (1999).
58. N. Arkani-Hamed, S. Dimopoulos and G. Dvali, "The Hierarchy problem and new dimensions at a millimetre", *Physics Letters B* **429**, 263-272 (1998).
59. L. Randall and R. Sundrum, "A Large mass hierarchy from a small extra dimension", *Physical Review Letters* **83**, 3370-3373 (1999).
60. S. Pakvasa and J.W. Valle, "Neutrino Properties Before and After KamLAND", *Proc. Indian Natl. Sci. Acad.* **70A**, 189–222 (2004).
61. I. Antoniadis, "A Possible new dimension at a few TeV", *Physics Letters B* **246**, 377–384 (1990).
62. G. Dvali and A.Y. Smirnov, "Probing Large Extra Dimensions with Neutrinos", *Nuclear Physics B* **563**, 63–81 (1999).

63. N. Arkani-Hamed, S. Dimopoulos, G. Dvali and N. Kaloper, "Infinitely Large New Dimensions", *Physical Review Letters* **84**, 586–589 (2000).
64. E. Palti, "The Swampland: Introduction and Review", *Fortsch. Phys.* **67**, 1900037 (2019).
65. M. van Beest, J. Calderón-Infante, D. Mirfendereski and I. Valenzuela, "Lectures on the Swampland Program in String Compactifcations", *Physics Reports* **989**, 1 (2022).
66. M. Graña and A. Herráez, "The Swampland Conjectures: A Bridge from Quantum Gravity to Particle Physics", *Universe* 7, 273 (2021).
67. S.-J. Lee, W. Lerche and T. Weigand, "Emergent strings from infinite distance limits", *Journal of High Energy Physics* **02**, 190 (2022).
68. D. Harlow, B. Heidenreich, M. Reece and T. Rudelius, "Weak gravity conjecture", *Reviews of Modern Physics* **95**, 035003 (2023).
69. A. Castellano, A. Herraez and L.E. Ibanez, "Towers and hierarchies in the Standard Model from emergence in quantum gravity", *Journal of High Energy Physics* **10**, 172 (2023).
70. D. Jia, "Experience in quantum physics: toward a theory of everything", arXiv:2306.11549v1 [quant-ph] (2023).
71. K. Urbanowski, "Cosmological constant in a universe born in the metastable false vacuum state", *European Physical Journal C* **82**, 242 (2022).
72. J.B. Barbour and B. Bertotti, "Mach's Principle and the Structure of Dynamical Theories", *Proceedings of Royal Society A* **382**, 1783, 295–306 (1982).
73. E. Anderson, J.B. Barbour, B.Z. Foster, B. Kelleher and N.O' Murchadha, "The physical gravitational degrees of freedom," *Classical and Quantum Gravity* **22**, 1795–1802 (2005); e-print arXiv:gr-qc/0407104.
74. J.B. Barbour, B.Z. Foster, and N.O' Murchadha, "Relativity without relativity," *Classical and Quantum Gravity* **19**, 3217–3248 (2002); e-print arXiv:gr-qc/0012089.
75. J.B. Barbour, "The Nature of Time", http://arxiv.org/abs/0903.3489 (2009).
76. J.B. Barbour, "The end of time: the next revolution in physics, Oxford University Press, Oxford (2000).
77. C. Rovelli, "Time in quantum gravity: an hypothesis", *Physical Review D* **43**, 2, 442-456 (1991).
78. C. Rovelli, "Quantum mechanics without time: a model", *Physical Review D* **42**, 8, 2638-2646 (1991).
79. C. Rovelli, "Analysis of the different meaning of the concept of time in different physical theories", *Il Nuovo Cimento B* **110**, 1, 81-93 (1995).
80. C. Rovelli, *Quantum Gravity*, Cambridge University Press (2004).

81. C. Rovelli, "Statistical mechanics of gravity and thermodynamical origin of time", *Classical and Quantum Gravity* **10**, 8, 1549-1566 (1993).
82. C. Rovelli, "The statistical state of the universe", *Classical and Quantum Gravity* **10**, 8, 1567-1578 (1993).
83. C. Rovelli, "Forget time", arXiv:0903.3832v3 [gr-qc] (2009).
84. A. Valdes-Hernandez, C.G. Maglione, A.P. Majtey and A.R. Plastino, "Entanglement and the ticking of the clock", *Revista Brasileira de Ensino de Fisica* **42**, e20190313 (2020).
85. D. Dolce, "Introduction to the quantum theory of elementary cycles: the emergence of space, time and quantum", in *Beyond Peaceful Coexistence: The Emergence of Space, Time and Quantum*, eds I. Licata and G. 't Hooft, World Scientific (2016), pp. 93-135.
86. D. Dolce, "New stringy physics beyond quantum mechanics from the Feynman path integral", DOI:10.21203/rs.3.rs-503118/v1, (2022).
87. P.S. Joshi, *Global Aspects in Gravitation and Cosmology*, Clarendon Press, Oxford (1993).
88. S.W. Hawking, "Particle creation by black holes", *Communications in Mathematical Physics* **43**, 199 (1975).
89. S.W. Hawking, "Breakdown of predictability in gravitational collapse", *Communications in Mathematical Physics* **14**, 2460 (1976).
90. L. Susskind, L. Thorlacius and J. Uglum, "The Stretched horizon and black hole complementarity", *Physical Review D* **48**, 3743 (1993).
91. L. Susskind, "String theory and the principles of black hole complementarity", *Physical Review Letters.* **71**, 2367 (1993).
92. L. Susskind and L. Thorlacius, "Gedanken experiments involving black holes", *Physical Review D* **49**, 966 (1994).
93. D.N. Page, "Information in black hole radiation", *Physical Review Letters* **71**, 3743 (1993).
94. P. Hayden and J. Preskill, "Black holes as mirrors: Quantum information in random subsystems", *Journal of High Energy Physics* **09**, 120 (2007).
95. A. Almheiri, D. Marolf, J. Polchinski, and J. Sully, "Black holes: complementarity or firewalls?", *Journal of High Energy Physics* **62** (2013).
96. S.W. Hawking, "Information preservation and weather forecasting for black holes", arXiv:1401.5761 (2014).
97. C. Vaz, "Black holes as gravitational atoms", *International Journal of Modern Physics D* **23**, 1441002 (2014).
98. C. Corda, "Black hole spectra from Vaz's quantum gravitational collapse", arXiv:2305.02184v2 [gr-qc] (2023).
99. R. Bousso and G. Penington, "Islands far outside the horizon", arXiv:2312.03078v3 [hep-th] (2023).

100. N. Engelhardt and A.C. Wall, "Decoding the apparent horizon: a coarse-grained holographic entropy", arXiv:1706.02038v4 [hep-th] (2019).

101. B. Concepcion, Y. Nomura, K. Ritchie and S. Weiss, "Complementarity for a dynamical black hole", *Physical Review D* **110**, 086002 (2024).

102. P. Hayden and J. Preskill, "Black holes as mirrors: Quantum information in random subsystems", *Journal of High Energy Physics* **09**, 120 (2007).

103. W. G. Unruh and R. M. Wald, "Acceleration radiation and generalized second law of thermodynamics", *Physical Review D* **25**, 942 (1982).

104. A. R. Brown, "Tensile strength and the mining of black holes", *Physical Review Letters* **111**, 211301 (2013).

105. Y. Nomura, "Reanalyzing an evaporating black hole", *Physical Review D* **99**, 086004 (2019).

106. W. Kim and M. Nam, "Investigation of black hole complementarity in ADS_2 black holes", arXiv:2310.13274v2 [hep-th] (2024).

107. J. Maldacena and L. Susskind, "Cool horizons for entangled black holes", *Fortschritte der Physik* **61**, 9, 781–811 (2013).

108. D. Marolf and H. Maxfield, "Observations of Hawking radiation: The Page curve and baby universes", *Journal of High Energy Physics* **04**, 272 (2021).

109. D. Jia, "Modes of experience in a superposed world", arXiv:2208.10920, 10.48550/ arxiv.2208.10920 (2022).

HADRONIC JOURNAL 48, 185 - 198 (2025)
DOI 10.29083/HJ.48.02.2025/SC185

MASSES OF MODIFIED ATOMS OF TUNGSTEN ISOTOPES

Mikhail Kashchenko
Ural Federal University, Yekaterinburg, Russia
Ural State Forestry University, Yekaterinburg, Russia
mpk46@mail.ru
Maksim Kovalenko
Ural Federal University, Yekaterinburg, Russia
Vladimir Pechorsry
Ural State Forestry University, Yekaterinburg, Russia
Nadezhda Kashchenko
Ural Federal University, Yekaterinburg, Russia

Received February 17, 2025

Abstract

The mass spectra of modified tungsten atoms with increased masses, detected on a titanium anode during arc discharges in air using a tungsten cathode, are presented. The calibration of the spectra is discussed in sufficient detail. The results indicate the existence of massive compact electron (ee)-pairs that change the chemical properties of atoms by reducing the effective charge of the nucleus when (ee)-pairs are included in the electron shell, or when pseudonuclei are formed.

Keywords: hadron mechanics, (*ee*)-pairs, mass spectra, modified atoms, tungsten isotopes.

Copyright © 2025 by Hadronic Press, Inc., Palm Harbor, FL 34684, USA

The mass spectra of tungsten isotopes were presented in reports, and were partially presented in the preprint [6], but were not discussed in detail.
It is clear that experimental confirmation of the existence of massive (*ee*) -pairs is of fundamental importance for the implementation of low-temperature nuclear synthesis [2, 3, 4, 7], therefore these spectra are given below with a fairly detailed discussion of the calibration results, which ensures, in particular, the reliability of the data in Table 1.

Experimental details

Fig. 1 schematically shows an arc discharge in air between a tungsten cathode and a titanium anode. Three arc discharge channels (indicated by arrows) symbolize that the single channel that actually exists can change orientation and shape during the discharge. During the discharge, the cathode material is partially destroyed, and a coating containing tungsten atoms forms on the anode surface. Typical values of voltage U and current I are given, as well as the diameters of the electrodes and the gap between them.

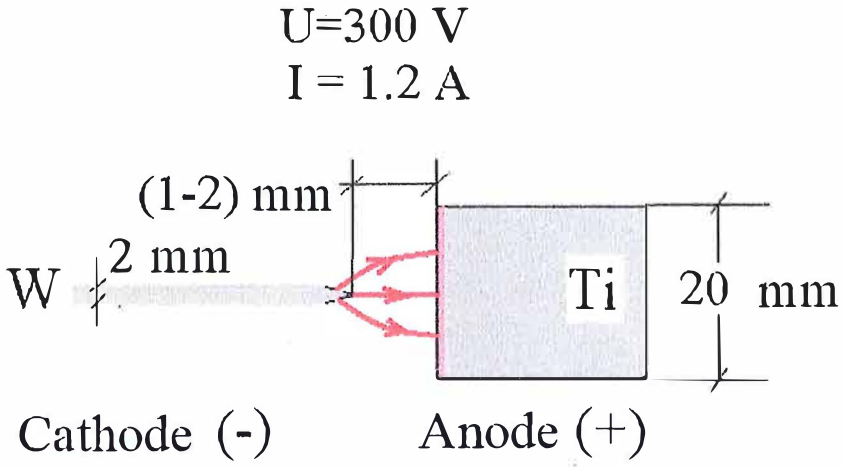


Fig. 1. Scheme of arc discharge in air environment

During an arc discharge in an air environment between a tungsten cathode and an anode made of different materials, the cathode material is

partially destroyed, and a coating containing tungsten atoms is formed on the anode surface. In this work, a scraping from the surface of a titanium anode in the form of a powder with characteristic sizes of powder particles of about 10 - 100 μm was studied. The resulting powder was pressed (pressed) into indium foil, placed in the analyzer chamber of the mass spectrometer, which was pumped out to a high vacuum of about 10^{-6} Pa.

The analysis method used was time-of-flight mass spectrometry of secondary ions in static mode (PHI TRIFT V nano TOF device). In the analytical gun, the primary gold ions have an energy of 30 keV. If it is necessary to clean the surface of the sample, sputtering with argon ions with an energy of 5 keV is used. The method is very surface-sensitive, in fact, several outer atomic layers are studied Primary short - term sputtering allows to remove the upper layer of sorbed gases, water and other possible surface contaminants. Further sputtering allows to study the composition in the volume of samples.

The typical mass resolution in the studied samples is $m/\Delta m \sim 4000 - 7000$, which allows peaks with a mass difference of ~ 0.02 amu to be distinguished. The spectra were calibrated using the most intense known peaks characteristic of the sample.

Results

An anomalous mass shift of tungsten isotopes, which arrived during the discharges from the tungsten cathode, was detected on the samples from the titanium anode. The general spectrum of the sample after primary sputtering is shown in Fig. 2.

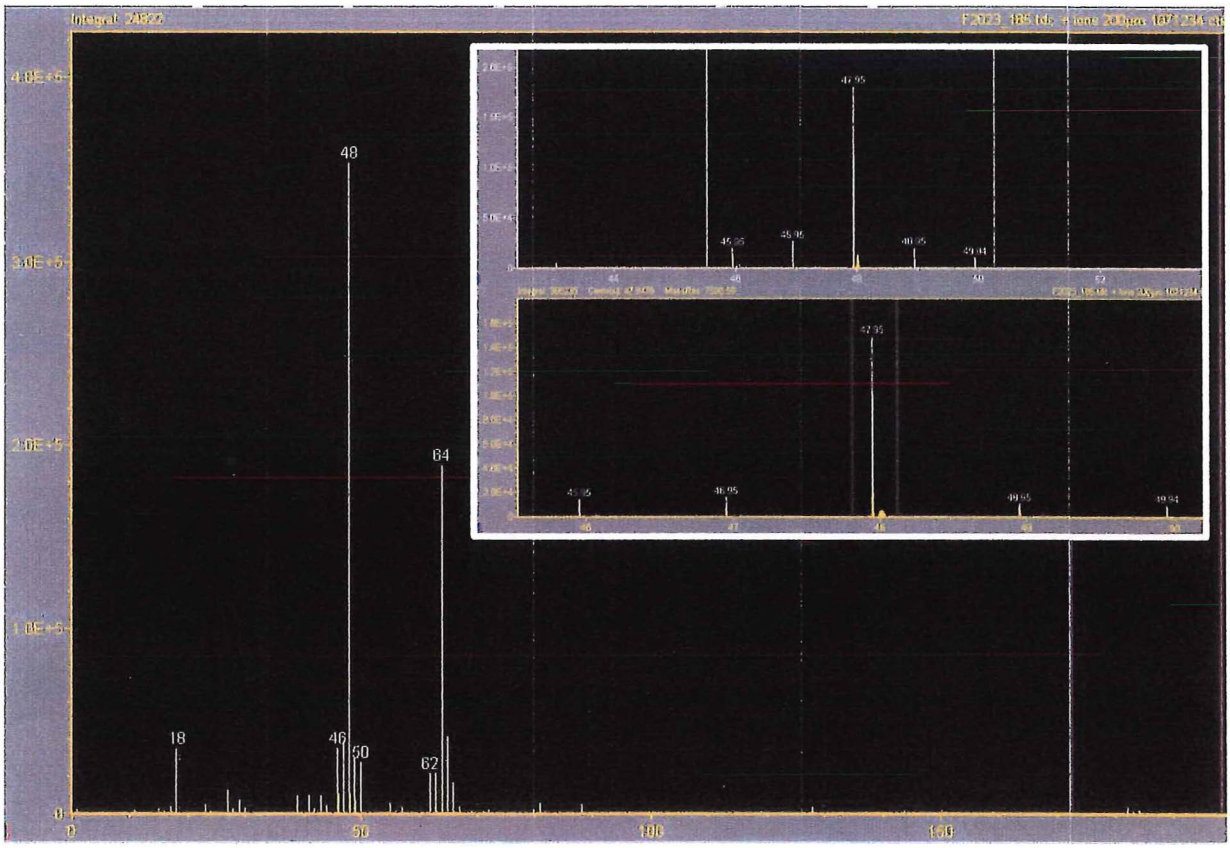


Fig. 2. General mass spectrum of titanium anode samples. The inset shows titanium isotopes.

The main peaks are titanium isotopes (titanium dominates - 48), titanium oxide

(peaks around 64 mass), water at 18 mass. The inset shows that the titanium isotopic composition is natural, the mass resolution is m /Δm = 7600.

The spectrum on these samples was calibrated using the peaks of hydrogen, ^{48}Ti, and oxide ^{48}TiO. The region of the mass spectrum including tungsten is shown in Fig. 3.

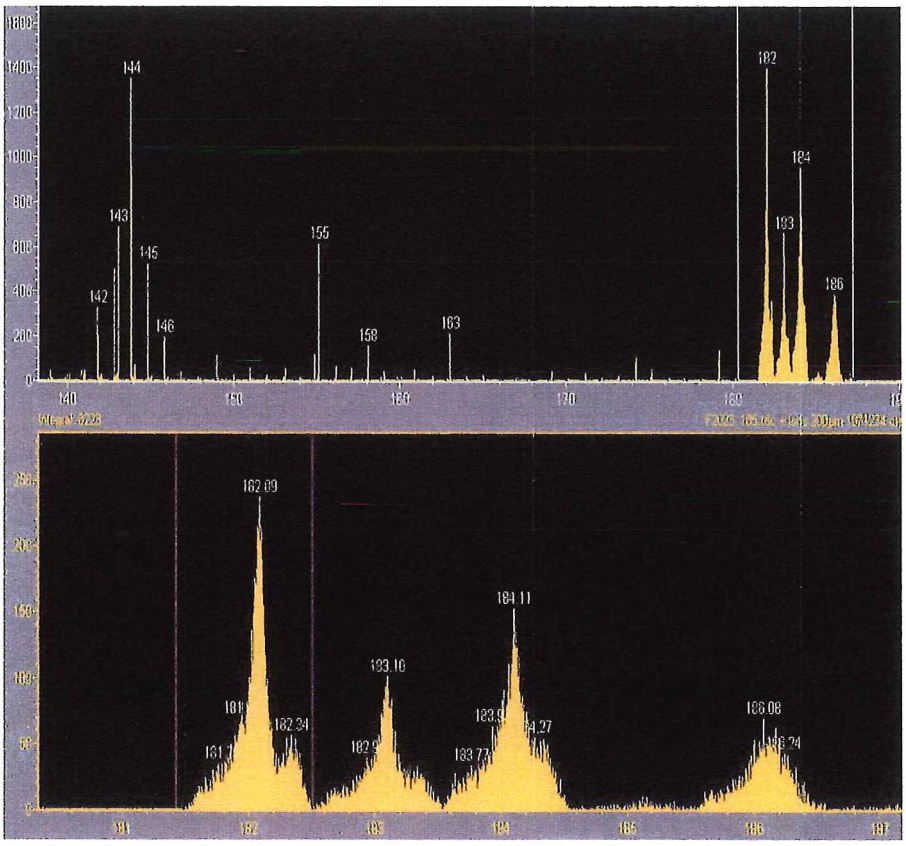


Fig. 3. Mass spectrum of modified tungsten in scrapings from the surface of a titanium anode.

From the data in Fig. 3 it is evident that the tungsten peaks are noticeably shifted from their natural positions towards larger masses, and are also very wide compared to the other peaks in the spectrum. For clarity, Fig. 4 shows the standard intensities and positions of the isotopes in the mass spectrum.

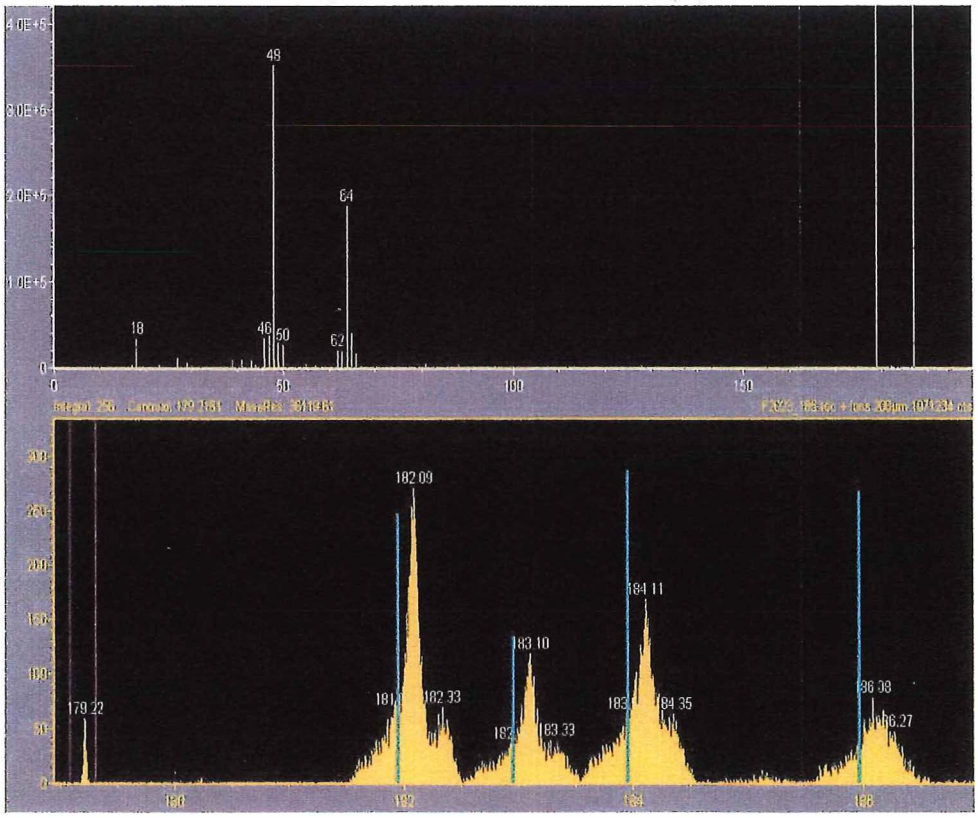


Fig. 4. Standard "natural" positions of tungsten isotopes in the mass spectrum (blue vertical lines), together with the actual signal.

The overall intensity of the tungsten signal is low, since tungsten is initially absent from the anode and flies in from the cathode during discharges. It is also worth noting the difference in the intensities of the isotopes of natural and modified tungsten. A typical spectrum of a tungsten cathode (a sample in the form of an electrode polishing), after conducting experiments with discharges and after short-term argon sputtering, is shown in Fig. 5. Note that the tungsten cathode contained an admixture of lanthanum

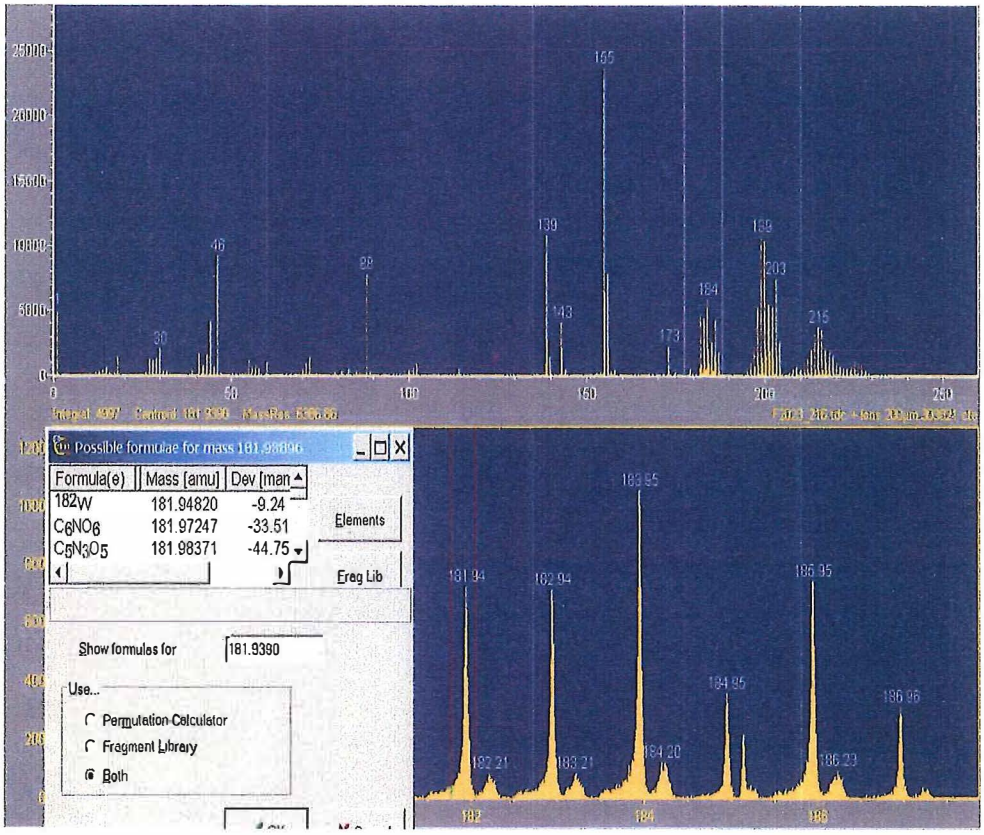


Fig. 5. Mass spectrum of a tungsten cathode. At the bottom is the mass region of tungsten isotopes, and the deviation of the ^{182}W peak from the standard position.

The spectrum in Fig. 5 was calibrated using hydrogen, lanthanum, and lanthanum oxide (139 and 155 masses). The deviations of the tungsten isotope positions from the reference values in the mass spectrum were less than 0.01 amu. Tungsten oxides are also clearly visible (a group of peaks at masses of ~ 200 and 216). WH hydrides are also visible (e.g., at 185 and 187 amu), which must be taken into account when determining the isotopic composition. Taking them into account, the isotopic composition can be considered natural (standard).

Discussion of results

First of all, we assume that the clearly expressed small "peaks" to the left of the main peaks in the lower part of Fig. 5 correspond to the masses m* of the modified atoms (containing (*ee*) – pairs) in contrast to the initial masses m of the tungsten atoms. The data in Table 1 correspond to one of the previously performed measurements and are close to the values indicated in Fig. 5.

The nature of the anomalous broadening to the left of the main peaks (see the lower part of Fig. 5) is due, according to [6], to small changes in the ion velocities that occur during the process of electrons escaping from the shell of an atom during the formation of a massive (*ee*) - pair in the shell. That is, this fact also indirectly testifies in favor of the existence of (*ee*) - pairs.

It should be noted that the anomalous shift (see Fig. 4) in the masses of tungsten atoms that reached the anode is not always observed. The use of copper or tantalum-based alloy as anodes was not accompanied by such an effect. With an aluminum anode, both normal and modified tungsten spectra were observed. Apparently, this effect is associated, on the one hand, with a decrease in the binding energy of modified tungsten atoms with the cathode (in accordance with the data in Table 2). On the other hand, the processes of bombardment of the cathode by atoms of the anode substance are also important. Moreover, first of all, the modified atoms of the anode will reach the cathode if the value of Q for them is less than that of the initial atoms of the anode. From the data of Table 3, in particular, it is clear that the values of Q for the mentioned metals differ.

Table 3. Molar heats of vaporization Q for a number of elements

Element Q (kJ/mol)	Al 284,1	Ti 422,6	Cu 304,6	Ta 758
Элемент* (n=1) Q (kJ/mol)	Al*→ Na 97,9	Ti*→Ca 153,6	Cu*→Co 389,1	Ta*→La 402

Элемент** (n=2) Q (kJ/mol)	Al**→ F 6,544	Ti**→Ar 6,45	Cu**→Mn 21	Ta**→Cs 68.3

Thus, for Al* and Ti* atoms, the heat of evaporation is significantly less than for Cu* and Ta*. In addition, for more definite conclusions, control over the value of interelectrode distances and corresponding voltages is required.

It is natural to assume that the tungsten atoms with shifted masses on the titanium anode correspond to the small «peaks» of modified atoms W* in Fig. 5 and, with better resolution, the lower part of Fig. 4. In Fig. 6, part of Fig. 4 is presented with the inclusion of only four modified tungsten isotopes.

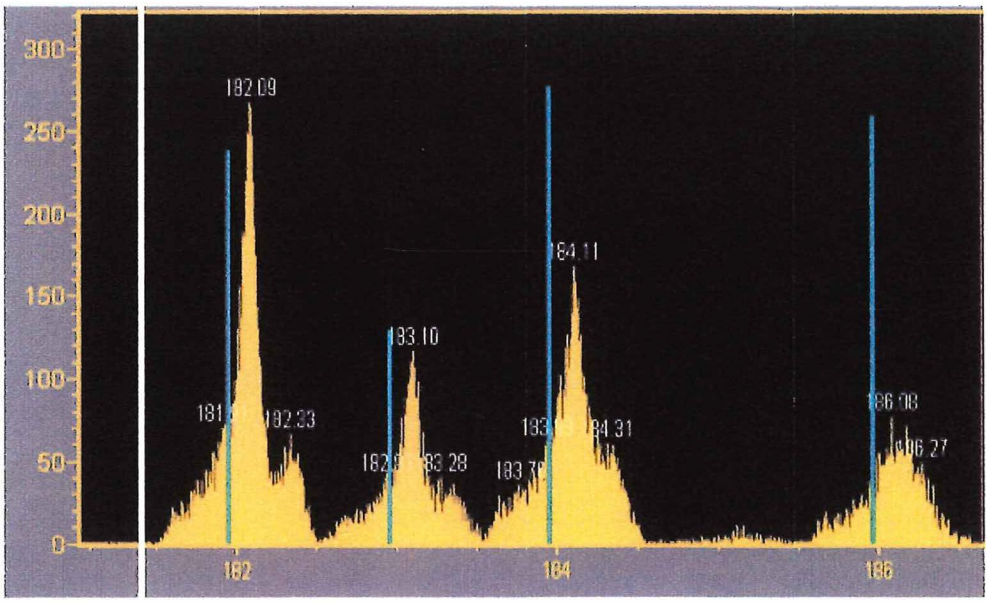


Fig. 6. Mass spectrum of tungsten isotopes W* on a titanium anode

The appearance of an additional small "peak" at 182.23 amu is noteworthy, which possibly corresponds to atoms with two (*ee*) - pairs. Similar mass spectrum inhomogeneities are present to a lesser extent in the vicinity of 183.28 amu and 184.28 amu. As can be seen from Fig. 6, sputtering for 30 seconds still preserves the above-mentioned spectrum inhomogeneities, but increasing the sputtering time by an order of magnitude practically eliminates them.

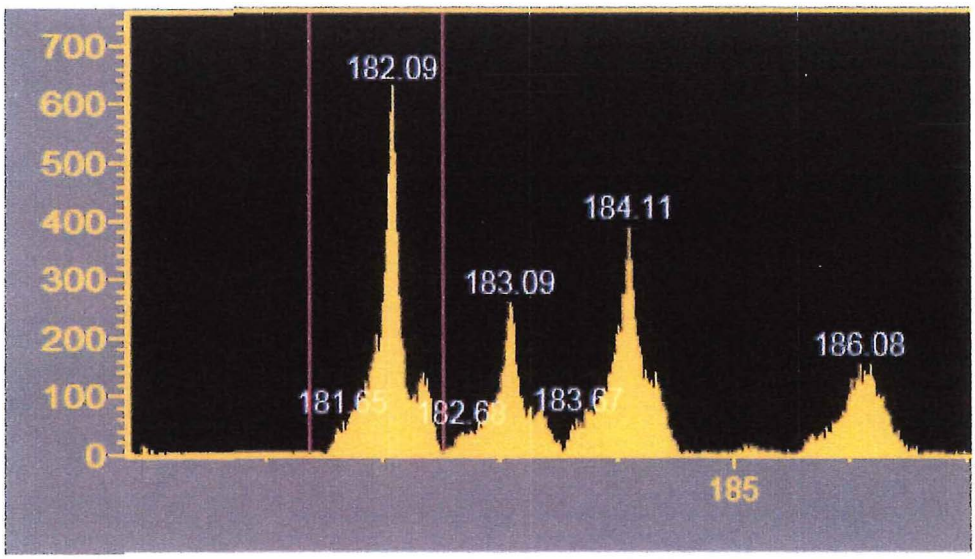


Fig. 7. Mass spectrum of tungsten isotopes W* on a titanium anode (after 30 sec of sputtering)

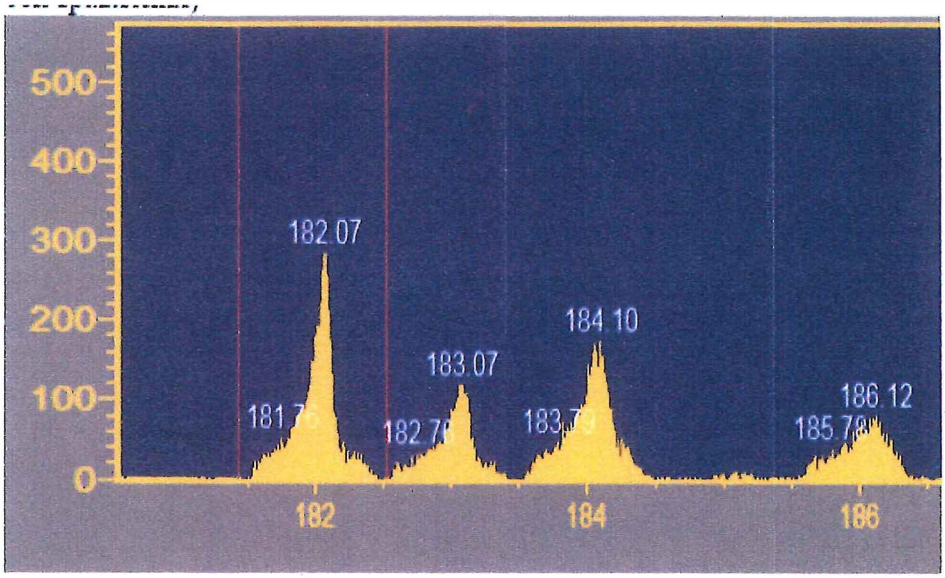


Fig. 8. Mass spectrum of tungsten isotopes W* on a titanium anode (after 300 sec sputtering)

Another remark concerns the question of the number of (*ee*) - pairs corresponding to the broadened small "peak". A comparative analysis of the mass spectra of titanium isotopes and their oxides [8] leads to the conclusion that each of the small "peaks" closest to the main peak on the right corresponds to atoms containing one (*ee*) - pair. The width of the small "peak" gives a spread in the mass values of individual (*ee*) - pairs. This conclusion is basically the same for tungsten isotopes, the examination of the mass spectra of whose oxides deserves a separate discussion.

The study of the mass spectra of atoms in the composition of scrapings from the cathode surface is also of interest.

The obtained results testify in favor of the existence of contact-bound massive electron pairs, which does not fit into the framework of quantum mechanics, but agrees with the conclusions of hadron mechanics, which takes into account the finite sizes of particles and generalizes quantum mechanics. The

authors interpret the modified atoms as atoms containing (*ee*)-pairs in deep orbits near the nucleus. However, according to the conclusions of hadron mechanics, a specific bound state of (*ee*)-pairs with protons is also admissible, leading to negatively charged pseudo-protons [3] and, accordingly, to pseudo-nuclei. Moreover, the neutron, in the spirit of Rutherford's idea, is considered as a bound state of an "isoproton" and an "isoelectron" [2]. The apparent contradiction with the bond imposed on the existence of such objects by the Heisenberg uncertainty relations is overcome in hadron mechanics by generalizing the uncertainty relations [9, 10] («isouncertainty» relations). Generalized uncertainty relations demonstrate an increase in the degree of determinism as the energy density of the medium increases. Thus, the experimental results indicating the appearance of modified atoms may also apply to the case of localization of (*ee*)-pairs in the region of nuclei (for example, in the composition of pseudoprotons).

Conclusion

The obtained experimental data testify in favor of the existence of massive electron pairs connected by contact interaction, as predicted by hadronic mechanics.

The emission of modified atoms from the surface of the tungsten cathode correlates well with the known fact of the appearance of "cathode spots" in local areas with increased current density.

References

1. Kashchenko M.P., Kovalenko M.A., Pechorsky V.I., Kupryazhkin A.Ya., Kashchenko N. Mass spectroscopic registration of modified tungsten isotopes with increased masses / The Fifteenth Internatinal Ural Seminar « Radiation damage physics of metals and alloys. Book of abstracts. Kyshtym, Russia, (2024) p. 30-31.
https://www.researchgate.net/publication/380169709

2. Santilli. R. M. *Foundations of Hadronic Chemistry with Applications to New Clean Energies and Fuels*. Boston - Dordrecht - London: Kluwer Academic Publishers. (2001) p. 554.
http://www.santilli-foundation.org/docs/Santilli-113.pdf
Russian translation by Aringazin, A. K.
http://www.i-b-r.org/docs/Santilli-Hadronic-Chemistry.pdf

3. Santilli R. M. Apparent Experimental Confirmation of Pseudoprotons and Their Application to New Clean Nuclear Energies // International Journal of Applied Physics and Mathematics. (2019) Vol. 9, № 2. p. 72–100.
http://www.santilli-foundation.org/docs/pseudoproton-verification-2018.pdf

4. Kashchenko M.P., Kashchenko N. Low-temperature Nuclear Fusion: an Introduction to the Problem and its Conceptual Solution, Ural State Forestry University, Yekaterinburg, (2022) p. 180.

5. Kashchenko M.P., Kashchenko N.M. Massive electron pairs as possibleinitiators of the electroplastic effect / Actual issues of strength: Collectionof abstracts of the LXVII International Conference (Ekaterinburg, April 2, 2024) / editor-in-chief D.V. Zaitsev. Ekaterinburg: UGMU Publishing House, (2024) p. 41 – 42.

6. Kashchenko M., Kovalenko M., Pechorsky V., Kashchenko N. Mass spectra of modified tungsten isotopes.
https://www.researchgate.net/publication/385193135
DOI: 10.13140/RG.2.2.22037.95206

7. Kashchenko M.P., Kashchenko N. Interpretation of the mechanism of the influence of π^0-mesons on the process of attraction of nuclei //Hadronic Journal. (2024) Vol. 47, p. 89-94.

8. Kashchenko M., Kovalenko M., Pechorsky V., Kashchenko N. Mass spectra of titanium isotopes and their oxides and estimation of the number of massive electron pairs in modified titanium atoms. https://www.researchgate.net/publication/380324916.
DOI: 10.13140/RG.2.2.36243.08488

9. Santilli R. M. Studies on the classical determinism predicted by A. Einstein, B. Podolsky and N. Rosen // Ratio Mathematica. (2019) Vol. 37, p. 5-23. http://www.eprdebates.org/docs/epr-paper-ii.pdf

10. Muktibodh A. Santilli's Recovering of Einstein's Determinism. Progress in Physics. (2024) p. 26-34.
http://www.eprdebates.org/docs/muktibodh-2024.pdf

HADRONIC JOURNAL 48, 199 - 206 (2025)
DOI 10.29083/HJ.48.02.2025/SC199

THERMODYNAMICS IN n-SPATIAL DIMENSIONS

J. J. Bevelacqua
Bevelacqua Resources
7531 Flint Crossing Circle SE
Owens Cross Roads, AL 35763
bevelresou@aol.com

Received April 8, 2025

Abstract

Conventional thermodynamics is well established, and is usually applied to three-dimensional space. Thermodynamics has an impact on astrophysical phenomena through conventional physics including the general theory of relativity. Accordingly, extensions of thermodynamics to multiple dimensions may also have astrophysical implications including the understanding of black holes dynamics, inflation following the big bang, and the evolution of the universe. This paper reviews the extension of thermodynamics to multiple spatial dimensions by reviewing a number of basic concepts including black body radiation; internal energy, heat capacity, entropy; and the ratio of specific heats (C_p/C_v).

KEYWORDS: Thermodynamics in n-spatial dimensions; astrophysical implications; black body radiation; ratio of specific heats (C_p/C_v); and internal energy, heat capacity, and entropy.

Copyright © 2025 by Hadronic Press, Inc., Palm Harbor, FL 34684, USA

1.0 Introduction

Thermodynamics and statistical mechanics in conventional spacetime (three spatial dimensions (n) and a time dimension) are well defined. For example, Newtonian mechanics and general relativity yield orbits with specific characteristics that are observed in physical structures including the Solar system. In a previous paper, Newtonian orbital results were generalized to n-dimensions ($n > 3$) with results that deviated from conventional orbits and associated characteristics [1].

In a similar manner, n-dimensional thermodynamics could yield results that differ from conventional thermodynamics and statistical mechanics [2-9]. A portion of these possible differences are addressed in this paper with an emphasis on the temperature dependence of selected physical quantities.

As an initial investigation of multidimensional thermodynamics, selected thermodynamic relationships are evaluated to determine their n-dimensional counterparts. For example, in conventional spacetime, (1) the energy density of black body radiation depends on the absolute temperature (T) that varies as T^4, (2) the specific heat of solids varies as T^3, and (3) the specific heat ratio (C_p/C_v) for a monatomic ideal gas is 5/3 [2-9].

Additional aspects of conventional thermodynamics and its applications are provided by Santilli [10-11]. Specific thermodynamic applications of Santilli's approach [10-11] are provided in [12].

In general, these thermodynamic relationships could depend on the dimensionality of spacetime, particularly the number of spatial dimensions. The generalized n-dimensional thermodynamic relationships, including statistical mechanics, have potential astrophysical importance associated with black hole thermodynamics, entropy evolution following the big bang, and inflation.

2.0 Assumptions

Given the theoretical nature of this paper, the assessment of the effects of multiple dimensions on selected thermodynamic quantities must rely on a number of assumptions. These assumptions include, but are not limited to:

1. Thermodynamic relationships in multiple dimensions have a similar physical form as their conventional spacetime counterparts, and must yield the expected three-dimensional results when $n = 3$.
2. The extra dimensions do not interact with each other or with conventional spacetime.
3. Specific characteristics of the extra dimensions are not considered.

4. The scale of the extra dimensions is not specified.
5. The specific spacetime topology, including the character of any compactified dimensions, evolves in a manner that is consistent with the aforementioned assumptions.
6. Any interaction of astrophysical phenomena (e.g., black body radiation) with the extra dimensions that could affect the character of these dimensions is not considered.
7. The number of polarization states allowed for photons in n dimensions is unknown. This uncertainty is absorbed into the C(n) term in (8).
8. All dimensions are independent.

The aforementioned assumptions are utilized in subsequent discussion.

3.0 Thermodynamic Relationships

In order to assess the theoretical effects of the postulated extra dimensions, specific thermodynamic quantities are evaluated. These include (1) black body radiation, (2) internal energy, heat capacity, and entropy, and (3) C_p/C_v ratio.

3.1 Black Body Radiation

In conventional spacetime, the energy density of blackbody radiation depends on temperature. The energy density of the output of a blackbody radiator scales as the absolute temperature (T^4). A general relationship for extending conventional thermodynamics to multiple dimensions must rely on a number of assumptions noted previously.

The first and second laws of thermodynamics leads to an internal energy equation [2]

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P \qquad (1)$$

where the parameters in Eq. 1 refer to the blackbody radiation. In Eq. 1, U is the total internal energy, V is the volume, T is the absolute temperature, and P is the pressure. The total energy is related to the energy density (ρ)

$$U = \rho V \qquad (2)$$

In conventional spacetime, the pressure of blackbody radiation is related to the density [8]

$$P = \rho/3 \tag{3}$$

In multiple dimensional spacetime, Eq. 3 will in general depend on the number of spatial dimensions. In this paper, it is additionally assumed that the blackbody radiation in n dimensions follows from the conventional spacetime relationship of (3)

$$P = \rho/n \tag{4}$$

(1), (2), and (4) lead to the general result for n-dimensional spacetime

$$\rho = T\frac{1}{n}\frac{d\rho}{dT} - \frac{\rho}{n} \tag{5}$$

Eq. 5 simplifies to the result

$$\frac{d\rho}{\rho} = (n+1)\frac{dT}{T} \tag{6}$$

that has the solution that can be written in the form

$$\rho = CT^{n+1} \tag{7}$$

where C is a constant of integration. A similar result can be derived in an alternative manner utilizing statistical mechanics [13,14]. (7) leads to the expected result that in three spatial dimensions the black body radiation intensity varies as T^4.

3.2 Internal Energy, Heat Capacity, and Entropy

In the previous section, a thermodynamic approach utilized the internal energy to address black body radiation. This section utilizes a statistical mechanics approach to perform an internal energy calculation, and to extend the results to determine of the heat capacity and entropy in n-dimensions. This is accomplished by generalizing conventional thermodynamics and statistical mechanics. Assuming one state per unit lattice in n-dimensional k-space $(2\pi/L)^n$ for each polarization, the density of states is

$$D(\omega)d\omega = \left[\frac{(2\pi)^{n/2}}{\Gamma\left(\frac{n}{2}\right)}\right]\left(\frac{L}{2\pi}\right)^n \omega^{n-1}dk = C(n)\omega^{n-1}d\omega \tag{8}$$

where the C(n) term includes the volume of an n-1 dimensional sphere, the number of states per unit lattice in k-space, and any polarization terms that involve an assumed linear dispersion relationship $\omega = v\,k$. The dispersion relationship relates the wave vector (k), velocity (v), and angular frequency (ω).

The total internal energy is defined by the density of states

$$U = \int D(\omega) d\omega \hbar\omega \langle n(\omega) \rangle = \int d\omega \hbar C(n) \omega^n \left(\frac{1}{exp(\hbar\omega/kT)-1} \right) \qquad (9)$$

where k is the Boltzmann constant, and $\langle n(\omega) \rangle$ is the average energy as a function of frequency. Using a change of variables $x = \hbar\omega/kT$, leads to a simplified result for the internal energy

$$U = B(n) T^{n+1} \int \left(\frac{x^n}{e^x - 1} \right) dx \qquad (10)$$

where the constant B(n) is related to C(n) multiplied by the terms generated by the change of variables and the constants appearing in (9). (10) reduces to the expected three dimensional result that was also derived in Section 3.1.

In conventional spacetime, (10) is usually evaluated using the Debye approximation where the upper integration limit is the Debye frequency. This approximation may not be valid in an n dimensional space. Instead, the integrals of (9) and (10) are evaluated using an unrestricted upper integration limit

$$U = B(n) T^{n+1} \int_0^\infty \left(\frac{x^n}{e^x - 1} \right) dx \qquad (11)$$

(11) has the solution

$$U = B(n) T^{n+1} \Gamma(n+1) \varsigma(n+1) = J(n) T^{n+1} \qquad (12)$$

where $\Gamma(n)$ is a gamma function of order n, $\zeta(n)$ is a Riemann zeta function of order n, and $J(n) = B(n)\,\Gamma(n+1)\,\zeta(n+1)$.

(12) permits the calculation of the heat capacity (C) using standard thermodynamic relations

$$C = \frac{\partial U}{\partial T} = J(n)(n+1) T^n \qquad (13)$$

In a similar manner, the entropy (S) is

$$S = \int \frac{C}{T} dT = J(n) \frac{n+1}{n} T^n + a \tag{14}$$

where a is a constant of integration that is to be determined based on observations.

3.3 C_p/C_v Ratio

The equipartition theorem suggests that the specific heat at constant volume (C_v) is

$$C_v = \frac{dg}{2} \tag{15}$$

where d is the number of degrees of freedom for the species and g is a constant. For a monatomic molecule in an n-dimensional space d = n. Accordingly.

$$C_v = \frac{ng}{2} \tag{16}$$

In addition, the heat capacities at constant volume and constant pressure (C_p) are related by the relationship [3]

$$C_p = C_v + g \tag{17}$$

(14) and (15) lead to the result

$$\frac{C_p}{C_v} = \frac{n+2}{n} \tag{18}$$

(18) provides the expected result of 5/3 for conventional spacetime.

4.0 Discussion

Thermodynamics has a significant impact on astrophysical phenomena [15-19]. For example, Hawking notes that there is a strong relationship between black holes and thermodynamics [17]. The commonly derived relationships involving information flow, heat capacity, Hawking radiation, and entropy could require modification in an n-dimensional spacetime. Relevant conclusions regarding the

behavior of thermodynamic quantities associated with black holes could be modified if multiple dimensional spacetime emerges in the vicinity of these bodies [15-19].

Hawking [17] noted that black holes have negative specific heat. This result may change as a function of the dimensionality of spacetime, and the interaction of these dimensions with their environment. In addition, derived relationships based on conventional spacetime (e.g., thermal equilibrium or time reversibility) may also require modification. These considerations could affect current theoretical studies of black holes and their relationship to white holes, and assumed time-symmetric behavior.

Current results related to a variety of topics including entropy, information flow, and Hawking radiation should also be reexamined for the theoretical consequences in multidimensional spacetime. The implications of multidimensional thermodynamics on quantum field theory, quantum gravity, and general relativity are also uncertain and should be carefully evaluated.

5.0 Conclusions

The generalization of thermodynamics to n spatial dimensions results in relationships that are consistent with their three-dimensional counterparts. Specific relationships are provided for blackbody radiation, internal energy, heat capacity, entropy, and C_p/C_v ratio. These and other thermodynamic relationships when generalized to n spatial dimensions could have astrophysical significance associated with the big bang, inflation, black hole thermodynamics, and entropy flow.

References

1) J. J. Bevelacqua, Can Extra Dimensions be Detected from Planetary Orbital Observations?, Physics Essays **26**, 381-387 (2013).
2) F. W. Sears, An introduction to Thermodynamics, The Kinetic Theory of Gases, and Statistical Mechanics, Addison-Wesley, Reading, MA (1964).
3) R. Reif, Fundamentals of Statistical and Thermal Physics, McGraw-Hill Book Company, New York (1965).
4) A. Sommerfeld, Thermodynamics and Statistical Mechanics, Academic Press, New York (1967).
5) E. A. Desloge, Thermal Physics, Holt, Reinhart and Winston, New York (1968).
6) L. D. Landau and E. M. Lifshitz, Statistical Physics, Second Edition, Addison-Wesley, Reading, MA (1969).

7) D. J. Amit and Y. Verbin, Statistical Physics- An Introductory Course, World Scientific, Singapore (2006).
8) S. Gasiorowics, The Structure of Matter: A Survey of Modern Physics, Addison-Wesley, Reading, MA (1979).
9) K. Huang, Statistical Mechanics, Second Edition, John Wiley & Sons, New York (1987).
10) R. M. Santilli, HADRONIC MATHEMATICS, MECHANICS AND CHEMISTRY, Volume I: Limitations of Einstein's Special and General Relativities, Quantum Mechanics and Quantum Chemistry, Institute for Basic Research, Palm Harbor, FL (2008).
11) R. M. Santilli, HADRONIC MATHEMATICS, MECHANICS AND CHEMISTRY, Volume II: Isodual Theory of Antimatter Antigravity and Spacetime Machines, Institute for Basic Research, Palm Harbor, FL (2008).
12) J. Dunning-Davies, The Thermodynamics Associated with Santilli's Hadronic Mechanics, Progress in Physics **4**, 24-26 (2006).
13) T. R. Cardoso and A. S. de Castro, The Blackbody Radiation in D-Dimensional Universes, Rev.Bras.Ens.Fis. **27,** 559-563 (2005).
14) H. Alnes, F. Ravndal, and I. K. Wehus, Black-body radiation in extra dimensions, arXiv:quant-ph/0506131v1 16 Jun 2005.
15) J. D. Bekenstein, Black holes and the second law, Lett. Nuovo Cim. **4**, 737-740 (1972).
16) J. D. Bekenstein, Black holes and entropy, Phys. Rev. D 7, 2333-2346 (1973).
17) S. W. Hawking, Black holes and thermodynamics, Phys, Rev. D **13**, 191-197 (1976).
18) R. M. Wald, The Thermodynamics of Black Holes, Living Rev. Relativity, 4, (2001), 6. [Online Article]: cited on <March 17, 2025e>, http://www.livingreviews.org/Articles/Volume4/2001-6wald/The thermodynamics of black holes, Living Rev. Rel. **4**, 6 (2001).
19) S. Carlip, Black Hole Thermodynamics, arXiv:1410.1486v2 [gr-qc] 26 Aug 2015.

HADRONIC JOURNAL 48, 207 - 230 (2025)

DOI 10.29083/HJ.48.02.2025/SC207

UNDERSTANDING THE ROLE OF NUCLEAR ELEMENTARY CHARGE AND ITS POTENTIAL IN ESTIMATING NUCLEAR BINDING ENERGY AND STRONG COUPLING CONSTANT

U. V. S. Seshavatharam
Honorary Faculty, I-SERVE, Survey no-42, Hitech City
Hyderabad-84, Telangana, India
seshavatharam.uvs@gmail.com
S. Lakshminarayana
Department of Nuclear Physics, Andhra University
Visakhapatnam-03, AP, India
lnsrirama@gmail.com

Received May 16, 2025
Revised May 29, 2025

Abstract

Considering our 4G model of final unification, there exists a nuclear elementary charge of magnitude *2.9464e* and strong coupling constant is the squared ratio of ordinary elementary charge to nuclear elementary charge. Nuclear elementary charge is having many applications in nuclear physics and other branches of physics like particle physics, super conductivity, condensed matter physics and unified physics. By refining the general nuclear binding energy formula, there is a possibility for defining a set of binding energy coefficients assumed to be linked with the strong coupling constant. By minimizing the error bars in the estimated binding energy, there is a chance for fixing the value of the strong coupling constant. Based on volume, surface, coulomb, asymmetry, pairing and congruent terms, it seems possible to fit the binding energy of isotopes of Z=1 to 137 and needs fine tuning for heavy isotopes of light proton numbers. Another interesting point is that, surface, coulomb and asymmetry energy coefficients can be considered as variable coefficients. Coulomb energy point of view, it is noticed that, increasing number of neutrons help in minimizing the nuclear radii against the coulomb repulsion. With reference to protons, neutrons and the coulomb energy, nuclear radii can be reviewed. With reference to an advanced binding energy formula and considering (2Z-1) and (3.5Z)+3 as the lower and upper mass limits, starting from Z=1 to 137, for the proposed 6 term semi empirical mass formula, RMS deviation is 3.25 MeV.

Keywords: 4G model of final unification; nuclear binding energy; nuclear radii; 6 term semi empirical mass formula; strong and electroweak mass formula; variable energy coefficients;

Copyright © 2025 by Hadronic Press, Inc., Palm Harbor, FL 34684, USA

1. Introduction

With reference to our 4G model of final unification, in our recent publications [1-12], we have developed a new formula for estimating nuclear binding energy [13-24] in terms of strong and electroweak interactions [25,26]. Our formula constitutes 4 simple terms and only one energy coefficient of magnitude 10.1 MeV. First term is a volume term, second term seems to be a representation of free nucleons associated with electroweak interaction, third term is a radial term and fourth one is an asymmetry term about the mean stable mass number. Considering this kind of approach, nuclear structure can be understood in terms of strong and weak interactions and complicated concepts like cold nuclear fusion can be understood in a theoretical approach positively.

In this short paper, we have presented a revised form of semi empirical mass formula (SEMF) with surface, coulomb and asymmetric energy coefficients as variables. This can be considered as a hybrid form of available SEMF. With a single set of energy coefficients, it seems to work for Z=1 to 137 and needs fine tuning for heavy isotopes of light proton numbers. In this context, we would like to emphasize the point that, strong coupling constant plays a vital role in fitting the nuclear binding energy coefficients. Clearly speaking, strong coupling constant can be inferred from nuclear binding energy coefficients. Considering our strong and electroweak mass formula and considering the proposed (revised) 6 term semi empirical mass formula, there is a chance to identify the strong coupling constant as a fundamental building block of atomic nuclei.

2. Three assumptions of 4G model of final Unification

Following our 4G model of final unification [1-12]

1) There exists a characteristic electroweak fermion of rest energy, $M_{wf}c^2 \cong 584.725$ GeV. It can be considered as the zygote of all elementary particles.
2) There exists a nuclear elementary charge in such a way that, $\left(\frac{e}{e_n}\right)^2 \cong \alpha_s \cong 0.1152 =$ Strong coupling constant and $e_n \cong 2.9464e$.
3) Each atomic interaction is associated with a characteristic large gravitational coupling constant. Their fitted magnitudes are,

$G_e \cong$ Electromgnetic gravitational constant $\cong 2.374335\times10^{37}$ $m^3kg^{-1}sec^{-2}$

$G_n \cong$ Nuclear gravitational constant $\cong 3.329561\times10^{28}$ $m^3kg^{-1}sec^{-2}$

$G_w \cong$ Electroweak gravitational constant $\cong 2.909745\times10^{22}$ $m^3kg^{-1}sec^{-2}$

It may be noted that,

1) Weak interaction point of view [25,26], following our assumptions, Fermi's weak coupling constant can be fitted with the following relations.

$$\left.\begin{aligned} G_F \cong \left(\frac{m_e}{m_p}\right)^2 \hbar c R_0^2 \cong G_w M_{wf}^2 R_w^2 \cong 1.44021\times10^{-62}\ \text{J.m}^3 \\ \text{where,}\ \begin{cases} R_0 \cong \dfrac{2G_n m_p}{c^2} \cong 1.24\times10^{-15}\ \text{m} \\ R_w \cong \dfrac{2G_w M_{wf}}{c^2} \cong 6.75\times10^{-19}\ \text{m} \end{cases} \end{aligned}\right\} \quad (1)$$

2) In a unified approach, most important point to be noted is that [10],

$$\hbar c \equiv G_w M_{wf}^2 \quad (2)$$

Clearly speaking, based on the electroweak interaction, the well believed quantum constant $\hbar c$ seems to have a deep inner meaning. Following this kind of relation, there is a possibility to understand the integral nature of quantum mechanics with a relation of the form, $n^2\hbar \equiv \dfrac{G_w\left(nM_{wf}\right)^2}{c}$ where $n = 1,2,3,..$ It needs further study with reference to EPR argument [27-30] and [1,31]. See section 5 for understanding the mystery of incompleteness of quantum mechanics in terms of hadronic mechanics [32-34]. String theory [35-39] can be made practical with reference to the three atomic gravitational constants associated with weak, strong and electromagnetic interaction gravitational constants. See Table 1. and Table 2. for sample string tensions and energies without any coupling constants.

Table.1 Charge dependent string tensions and string energies			
S.No	Interaction	String Tension	String energy
1	Weak	$\frac{c^4}{4G_w} \cong 6.94\times10^{10}$ N	$\sqrt{\frac{e^2}{4\pi\varepsilon_0}\left(\frac{c^4}{4G_w}\right)} \cong 24.975$ GeV
2	Strong	$\frac{c^4}{4G_n} \cong 6.065\times10^{4}$ N	$\sqrt{\frac{e_n^2}{4\pi\varepsilon_0}\left(\frac{c^4}{4G_n}\right)} \cong 68.79$ MeV
3	Electromagnetic	$\frac{c^4}{4G_e} \cong 8.505\times10^{-5}$ N	$\sqrt{\frac{e^2}{4\pi\varepsilon_0}\left(\frac{c^4}{4G_e}\right)} \cong 874.3$ eV

Table.2 Quantum string tensions and string energies			
S.No	Interaction	String Tension	String energy
1	Weak	$\frac{c^4}{4G_w} \cong 6.94\times10^{10}$ N	$\sqrt{hc\left(\frac{c^4}{4G_w}\right)} \cong 292.36$ GeV
2	Strong	$\frac{c^4}{4G_n} \cong 6.065\times10^{4}$ N	$\sqrt{hc\left(\frac{c^4}{4G_n}\right)} \cong 273.3$ MeV
3	Electromagnetic	$\frac{c^4}{4G_e} \cong 8.505\times10^{-5}$ N	$\sqrt{hc\left(\frac{c^4}{4G_e}\right)} \cong 10234.77$ eV

2.1 Understanding the electroweak coefficient and nuclear stability

Our basic idea is that, all the nucleons are not participating in the nuclear binding energy scheme and non-participating nucleons can be called as 'Free nucleons'. These free nucleons revolve round the nuclear core. Each free nucleon reduces the nuclear binding energy by 10.1 MeV. Protons and neutrons jointly play a crucial role in fixing the number of free nucleons. Electroweak interaction is having a key role in understanding free nucleons and nuclear stability against beta decay. In this context, we noticed that,

$$\frac{m_p}{M_w} \cong \left(\frac{\sqrt{(m_\pi c^2)^0 (m_\pi c^2)^\pm}}{\sqrt{(m_z c^2)^0 (m_w c^2)^\pm}}\right) \cong \left(\frac{\sqrt{134.98\times139.57}\ \text{MeV}}{\sqrt{91187.6\times80379.0}\ \text{MeV}}\right) \cong 0.0016032 \qquad (3)$$

Here ratio of rest mass of proton to the assumed electroweak fermion is equal to the ratio of mean mass of pions to the mean mass of electroweak bosons. Based on this unique and concrete observation, we are very confident to say that, strong and weak interactions play a vital role exploring the secrets of nuclear structure. Based on this electroweak

coefficient 0.0016, stability corresponding to nuclear beta decay can be understood with the following relation.

$$\left.\begin{aligned} A_s &\cong 2Z + 0.0016(2Z)^2 \cong 2Z + 0.0064Z^2 \\ &\rightarrow \frac{A_S - 2Z}{4Z^2} \cong 0.0016 \end{aligned}\right\} \qquad (4A)$$

One can find a similar relation in the literature [16]. This relation can be well tested for Z=21 to 92. For example,

$$\frac{45-(2\times 21)}{4(21)^2} \cong 0.00170; \quad \frac{63-(2\times 29)}{4(29)^2} \cong 0.00149; \quad \frac{89-(2\times 39)}{4(39)^2} \cong 0.00181;$$

$$\frac{109-(2\times 47)}{4(47)^2} \cong 0.0017; \quad \frac{169-(2\times 69)}{4(69)^2} \cong 0.00163; \quad \frac{238-(2\times 92)}{4(92)^2} \cong 0.001595;$$

This is one best practical and quantitative application of our proposed electroweak fermion and bosons. Following this relation and based on various semi empirical mass formulae [13-21], by knowing any stable mass number, its corresponding proton number can be estimated with,

$$Z \cong \frac{A_S}{1+\sqrt{1+0.0064A_S}} \cong \frac{A_S}{2+0.0153A_S^{2/3}} \qquad (4B)$$

$$\text{where } \frac{a_c}{2a_{asy}} \cong \frac{0.71\text{ MeV}}{2\times 23.21\text{ MeV}} \cong \frac{0.6615\text{ MeV}}{2\times 21.6091\text{ MeV}} \cong 0.0153$$

Considering this relation, we are working on understanding stable super heavy elements.

2.2 Revised strong and electroweak mass formula

Based on the liquid drop model, we present our revised strong and electroweak mass formula [1]. It needs a review for accuracy and we are working in this new direction. For Z=6 to 125,

$$BE \cong \left(A - A_{free} - A_{radial} - A_{asym}\right)\left(B_0 \cong 10.1\ \text{MeV}\right)$$

$$\cong \left\{ \begin{array}{l} A - \left\{ \frac{1}{2} + \left[\Upsilon_Z \left[\left(Z + Z^{2/3} + Z^{1/3}\right)^2 + \left(N + N^{2/3} + N^{1/3}\right)^2 + \left(\frac{Z}{N}\left(Z^2\right)\right)\right]\right]\right\} \\ -A^{1/3} - \left(\beta \times \frac{\left(A_s - A\right)^2}{A_s} \right) \end{array} \right\} 10.1\ \text{MeV} \qquad (5)$$

where, $A \times 10.1$ MeV represents the volume term

$A_{free} \times 10.1$ MeV represents the electroweak term

$A_{radial} \times 10.1$ MeV represents the radial term

$A_{asym} \times 10.1$ MeV represents the asymmetry term

$$\beta = 1 - \left(\frac{N - Z}{A} \right)^2$$

$$\Upsilon_Z \cong \Upsilon_0 \left[1 + \left(\Upsilon_0 Z \right) \right] \cong 0.000935\left[1 + 0.000935Z \right]$$

$$\text{where } \Upsilon_0 \cong \sqrt{\frac{e}{e_n}}\left(0.001605\right) \cong 0.000935$$

$$A_s \cong 2Z + 0.0064Z^2$$

$\cong$ Light house like (mean) stable mass number of Z.

$$B_0 \cong -\left(\frac{1}{\sqrt{\alpha_s}} \right) \frac{e^2}{8\pi\varepsilon_0 \left(\mathrm{h}/m_p c \right)} \cong -\frac{e_n^2}{8\pi\varepsilon_0 \left(G_n m_p / c^2 \right)} \cong -10.1\ \text{MeV}$$

where $\left\{ \begin{array}{l} \alpha_s = \text{Strong coupling constant} \cong 0.115 \text{ to } 0.12 \\ \mathrm{h}/m_p c = \text{Reduced Comption wavelength of proton} \\ G_n m_p / c^2 \cong 0.62 \times 10^{-15}\ \text{m} \end{array} \right.$

For evaluating the effectiveness of relation (5), we consider the following advanced relation as a reference [19].

$$BE \cong \left| \begin{array}{l} \left\{ \left[1 + \left(\frac{4k_v}{A^2} \right) |T_z| \left(|T_z| + 1 \right) \right] a_v * A \right\} + \left\{ \left[1 + \left(\frac{4k_s}{A^2} \right) |T_z| \left(|T_z| + 1 \right) \right] a_s * A^{\frac{2}{3}} \right\} \\ + \left\{ a_c * \left(\frac{Z^2}{A^{1/3}} \right) \right\} + \left\{ f_p * \frac{Z^2}{A} \right\} + E_p \end{array} \right| \qquad (6)$$

where, $T_z \cong$ 3rd component of isospin $= \frac{1}{2}(Z-N)$

$$\left\{\begin{array}{l} a_v = -15.4963 \text{ MeV}, a_s = 17.7937 \text{ MeV} \\ k_v = -1.8232, k_s = -2.2593 \\ a_c = 0.7093 \text{ MeV}, f_p = -1.2739 \text{ MeV} \\ d_n = 4.6919 \text{ MeV}, d_p = 4.7230 \text{ MeV} \\ d_{np} \approx -6.4920 \text{ MeV} \end{array}\right\} \text{ and}$$

$$\left\{\begin{array}{l} \text{for } (Z, N) \text{ Odd, } E_p \cong \frac{d_n}{N^{1/3}} + \frac{d_p}{Z^{1/3}} + \frac{d_{np}}{A^{2/3}} \\ \text{for } (\text{Odd } Z, \text{ Even } N), E_p \cong \frac{d_p}{Z^{1/3}} \\ \text{for } (\text{Even } Z, \text{ Odd } N), E_p \cong \frac{d_n}{N^{1/3}} \\ \text{for } (\text{Even } Z, \text{ Even } N), E_p \cong 0 \end{array}\right\}$$

3. Revised semi empirical mass formula having 6 terms

With reference to relation (6) and considering surface, Coulomb and asymmetry energy coefficients as variables, we present our revised 6 term semi empirical mass formula.

Volume energy coefficient, $a_v \cong 16.0$ MeV

Surface energy coefficient, $a_s \cong \beta \times 19.4$ MeV where $\beta \cong \left[1 - \left(\frac{N-Z}{A}\right)^2\right]$

Coulombic energy coefficient,

$$a_c \cong \left\{1 - \left(\frac{N-Z}{A}\right)^2\right\}^{-x} \times 0.71 \text{ MeV where } x \cong 0.75 - \frac{Z}{2A}$$

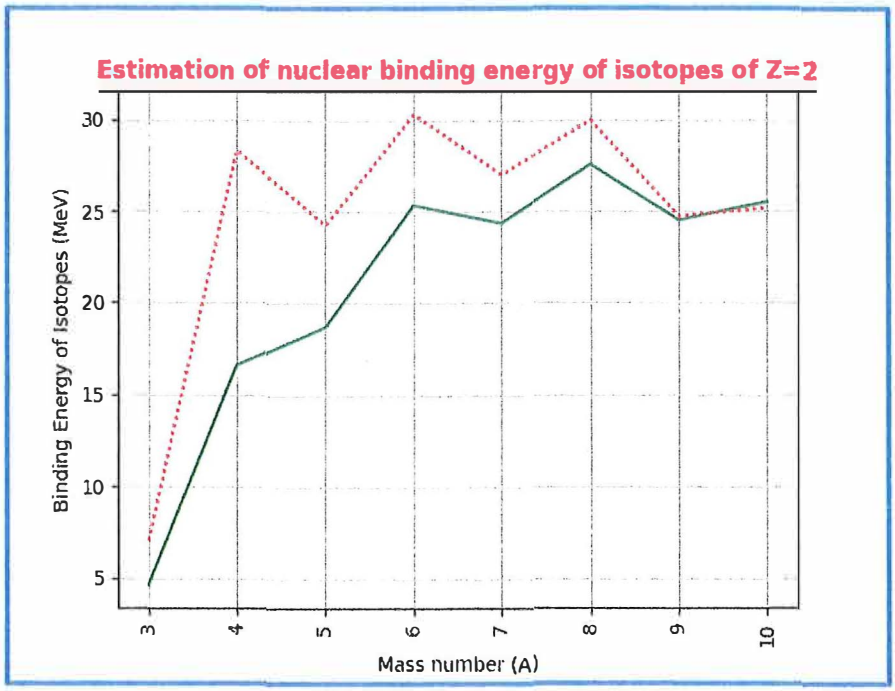
Asymmetry energy coefficient, $a_{asy} \cong \left[1 - \frac{1}{A}\right] \times 24.5$ MeV

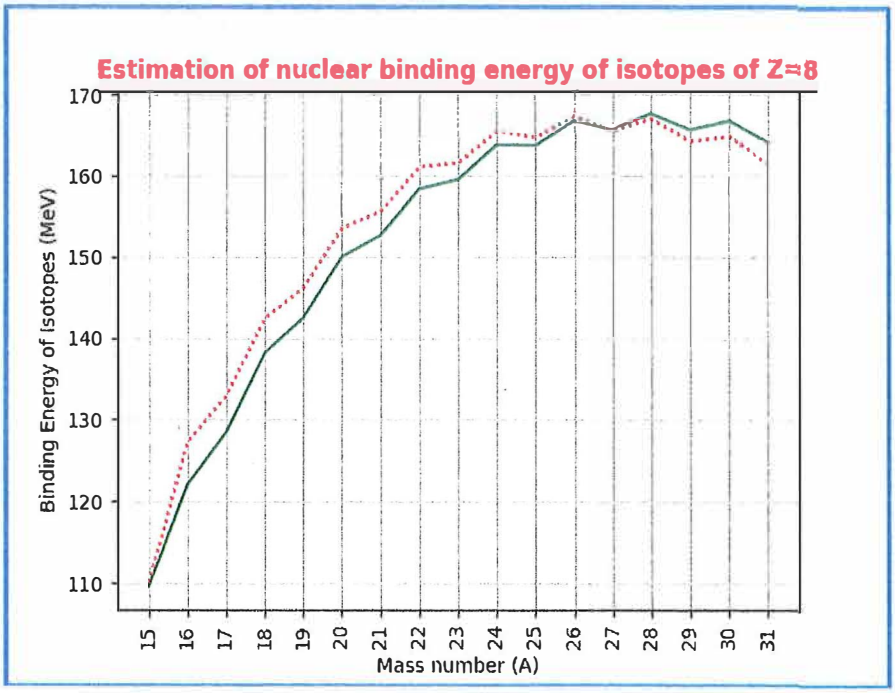
Pairing energy coefficient, $a_p \cong 10.0$ MeV

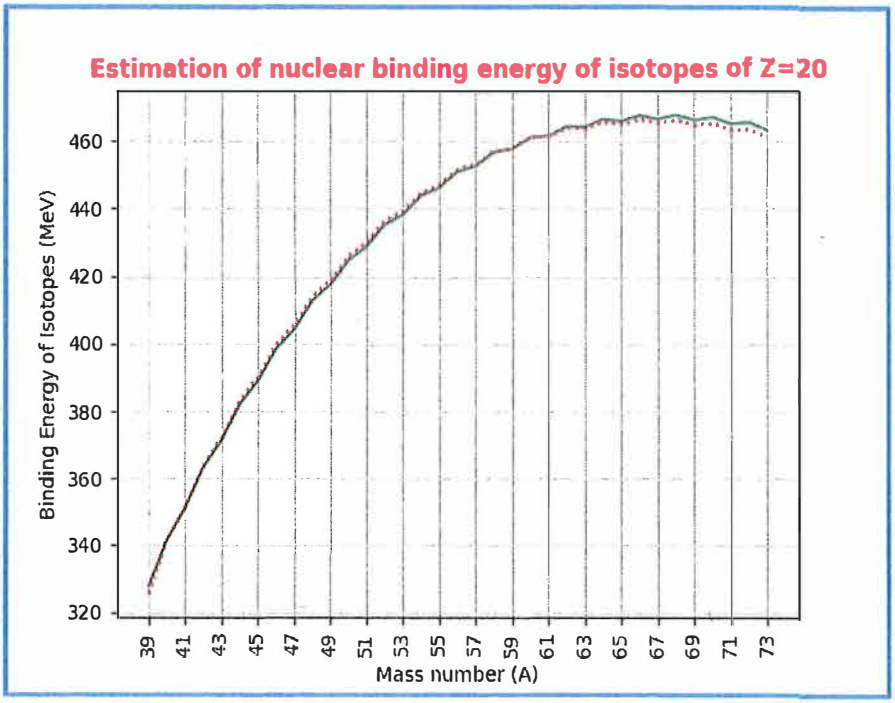
Congruent energy coefficient, $a_{cg} \cong 10.0$ MeV

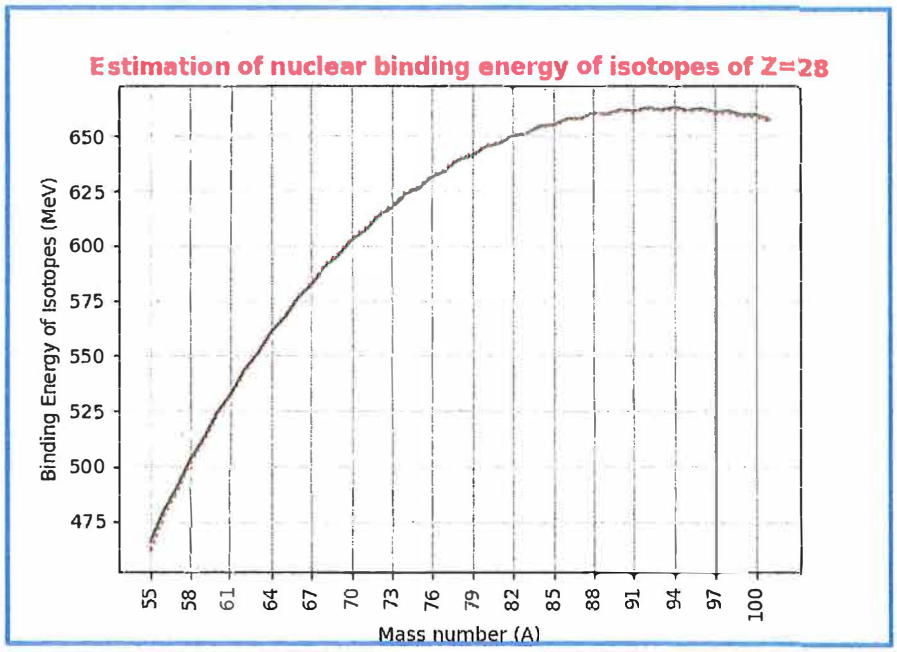
$$BE \cong \left[16.0\times A\right]-\left[\beta\times 19.4\times A^{2/3}\right]-\left[\frac{0.71\times Z^{2}}{\beta^{x}A^{1/3}}\right] \\ -\left\{\left[\left(1-\frac{1}{A}\right)24.5\right]\frac{(A-2Z)^{2}}{A}\right\}\pm\left[\frac{10}{\sqrt{A}}\right]+\left[10\times\exp\left(-4.2\frac{|N-Z|}{A}\right)\right] \quad (7)$$

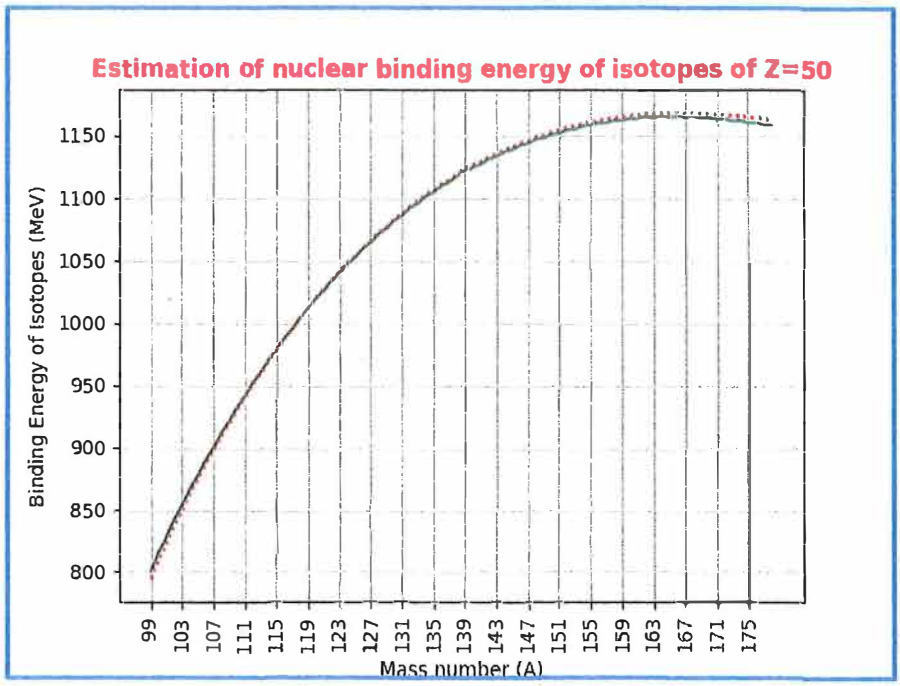
It seems to work from Z=1 to 137 without any difficulty and needs fine tuning for very heavy isotopes of very light proton numbers. See the following binding energy curves prepared for magic proton numbers [40], Z=2,8,20,28,50,82,100 and 114. Based on relation (6) and (7), red curve is our fit and green curve is the reference binding energy curve [19]. Interested readers can refer the nuclear binding energy data table as a supplementary material in our preprint, starting from Z=1 to 137 where lower and upper mass numbers are (2Z)-1 and (3.5Z)+3 respectively. (https://www.preprints.org/manuscript/202504.2431/v1). For our crude approximation, with reference to relation (6), for a wide range of protons and neutrons, relation (7) is having a root mean square deviation of 3.25 MeV.

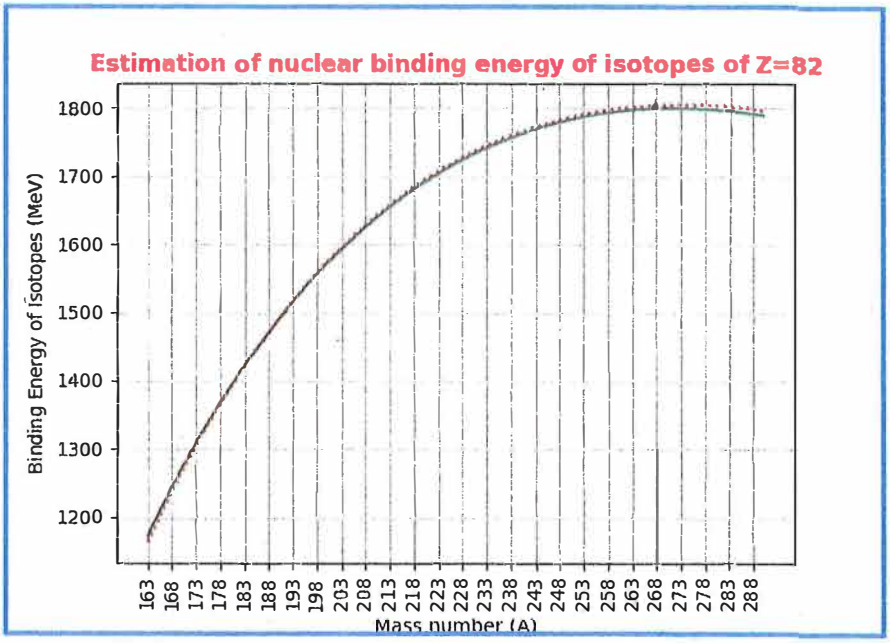


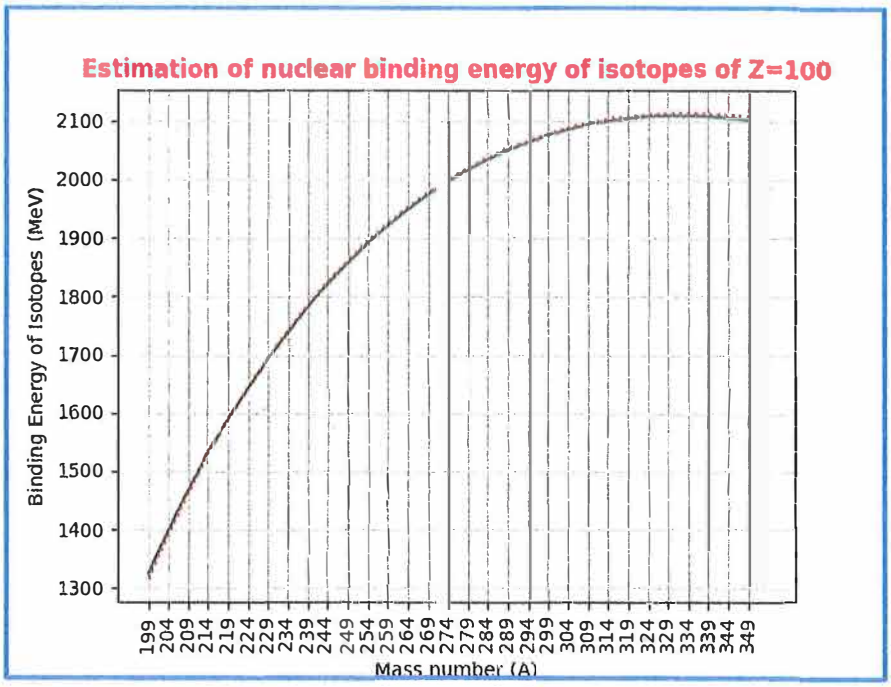

Estimation of nuclear binding energy of isotopes of Z=8
Binding Energy of Isotopes (MeV)
170
160
150
140
130
120
110
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
Mass number (A)

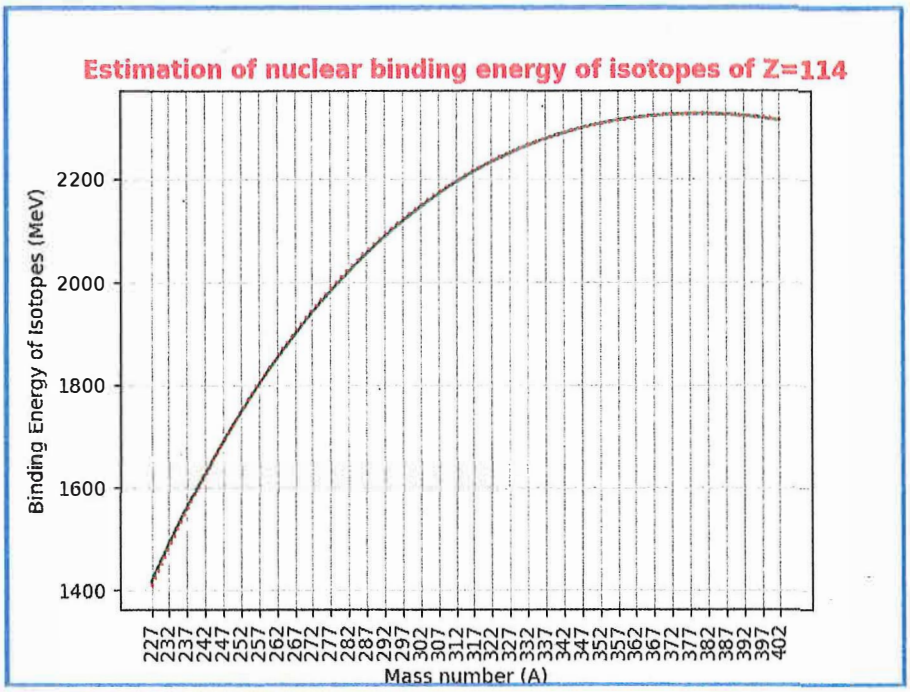

Estimation of nuclear binding energy of isotopes of Z=20
Binding Energy of Isotopes (MeV)
460
440
420
400
380
360
340
320
39
41
43
45
47
49
51
53
55
57
59
61
63
65
67
69
71
73
Mass number (A)


Estimation of nuclear binding energy of isotopes of Z=28
Binding Energy of Isotopes (MeV)
650
625
600
575
550
525
500
475
55
58
61
64
67
70
73
76
79
82
85
88
91
94
97
100
Mass number (A)


Estimation of nuclear binding energy of isotopes of Z=50
Binding Energy of Isotopes (MeV)
1150
1100
1050
1000
950
900
850
800
99
103
107
111
115
119
123
127
131
135
139
143
147
151
155
159
163
167
171
175
Mass number (A)


Estimation of nuclear binding energy of isotopes of Z=82
Binding Energy of isotopes (MeV)
1800
1700
1600
1500
1400
1300
1200
163 168 173 178 183 188 193 198 203 208 213 218 223 228 233 238 243 248 253 258 263 268 273 278 283 288
Mass number (A)


Estimation of nuclear binding energy of isotopes of Z=100
Binding Energy of Isotopes (MeV)
2100
2000
1900
1800
1700
1600
1500
1400
1300
199 204 209 214 219 224 229 234 239 244 249 254 259 264 269 274 279 284 289 294 299 304 309 314 319 324 329 334 339 344 349
Mass number (A)



4. Short discussion on the energy coefficients of the revised semi empirical mass formula

Let $e_n \equiv$ Nuclear elementary charge $\cong 2.9464e$.

$R_{pp} \cong \dfrac{G_n m_p}{c^2} \cong 0.61965\,\text{fm} \cong$ Characteristic physical radius of proton

where, $\begin{cases} G_n \cong \text{Nuclear gravitational constant} \\ m_p \cong \text{Rest mass of proton} \end{cases}$

Nuclear potential associated with e_n and R_{pp} can be expressed as,

$$PE \cong -\frac{e_n^2}{4\pi\varepsilon_0 R_{pp}} \cong \left(\frac{1}{\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_{pp}} \cong -20.174\,\text{MeV} \qquad (8)$$

where, $\alpha_s \cong$ Strong coupling constant [41]

With reference to both nucleons and considering a characteristic potential energy as,

$$(PE)_{np} \equiv -\frac{2e_n^2}{4\pi\varepsilon_0 R_{pp}} \cong \left(\frac{1}{\alpha_s}\right)\frac{2e^2}{4\pi\varepsilon_0 R_{pp}} \cong -40.35\,\text{MeV} \tag{9}$$

Based on this energy unit and beta decay, we consider the following relations for formulation of the energy coefficient. It needs further study.

1) Volume energy coefficient,

$$a_v \cong \left(\frac{m_e}{m_n - m_p}\right)\left(\frac{2e_n^2}{4\pi\varepsilon_0 R_{pp}}\right) \cong \left(\frac{1}{\alpha_s}\right)\left(\frac{1}{2.531}\right)\frac{2e^2}{4\pi\varepsilon_0 R_{pp}} \cong 15.94\,\text{MeV}$$

2) Asymmetry energy coefficient,

$$a_{asy} \cong \left(1-\frac{1}{2.531}\right)\left(\frac{2e_n^2}{4\pi\varepsilon_0 R_{pp}}\right) \cong \left(\frac{1}{\alpha_s}\right)\left(1-\frac{1}{2.531}\right)\frac{2e^2}{4\pi\varepsilon_0 R_{pp}} \cong 24.4\,\text{MeV}$$

3) Surface energy coefficient, $a_s \cong \dfrac{2a_v a_{asy}}{a_v + a_{asy}} \cong 19.3\ \text{MeV}$

4) Pairing energy coefficient, $a_p \cong \dfrac{e_n^2}{8\pi\varepsilon_0 R_{pp}} \cong \left(\dfrac{1}{\alpha_s}\right)\dfrac{e^2}{8\pi\varepsilon_0 R_{pp}} \cong 10.1\,\text{MeV}$

5) Congruence energy coefficient,

$$a_{cg} \cong \frac{e_n^2}{8\pi\varepsilon_0 R_{pp}} \cong \left(\frac{1}{\alpha_s}\right)\frac{e^2}{8\pi\varepsilon_0 R_{pp}} \cong 10.1\,\text{MeV}$$

6) Coulombic energy coefficient, $a_c \cong 0.7\,\text{MeV}$

Based on these energy coefficients, to a very good approximation, starting from Z=1 to 137,

$$BE \cong a_v A - \beta a_s A^{2/3} - a_c \frac{Z^2}{\beta^x A^{1/3}} - a_{asy}\frac{(A-2Z)^2}{A} \pm \frac{a_p}{\sqrt{A}} + a_{cg}\exp\left(-4.2\frac{|\mathrm{N}-\mathrm{Z}|}{\mathrm{A}}\right)$$

where $\beta \cong 1-\left(\dfrac{\mathrm{N}-\mathrm{Z}}{\mathrm{A}}\right)^2$ and

$$x \cong \left[0.75-\left(\frac{Z}{2A}\right)\right] \cong (0.5 \text{ to } 0.6) \rightarrow \text{Needs further study.} \tag{10}$$

Points of interest are:

1) Characteristic nuclear radii linked with coulomb energy and nuclear binding energy [4,17] can be expressed as,

$$R_{(Z,N)} \cong \left[1-\left(\frac{N-Z}{A}\right)^2\right]^x A^{1/3} \times R_0$$

$$\text{where} \begin{cases} R_0 \cong (1.24 \pm \text{to } 0.01)\,\text{fermi} \\ x \cong 0.75 - \dfrac{Z}{2A} \approx (0.5 \text{ to } 0.6) \end{cases} \tag{11}$$

Considering $\left(A^{1/3}R_0\right)$, increasing neutron number, reduces the Coulomb repulsion between protons and thus helps in minimizing the radius by a factor $\left[1-\left(\frac{N-Z}{A}\right)^2\right]^x$ where $x \cong 0.75 - \frac{Z}{2A}$.

2) Considering the Fermi gas model of the nucleus [42,43],

a) Characteristic Fermi energy can be expressed as

$$E_F \cong \left(\frac{2e_n^2}{4\pi\varepsilon_0 R_{pp}}\right) \cong \left(\frac{1}{\alpha_s}\right)\frac{2e^2}{4\pi\varepsilon_0 R_{pp}} \cong \left(\frac{4}{\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_0} \tag{12}$$

b) Volume and asymmetry energy coefficients can be expressed as,

$$\left.\begin{aligned} a_v &\cong \left(1-\frac{3}{5}\right)E_F \cong \left(1-\frac{3}{5}\right)\left(\frac{2e_n^2}{4\pi\varepsilon_0 R_{pp}}\right) \cong \left(\frac{4}{5\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_{pp}} \\ &\cong \left(\frac{8}{5\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_0} \cong \frac{\left(8\sqrt{3}\right)e^2}{4\pi\varepsilon_0 R_0} \quad \text{where} \left(\frac{1}{5\alpha_s}\right) \approx \sqrt{3} \end{aligned}\right\} \tag{13}$$

$$\left.\begin{aligned} a_{asy} &\cong \frac{3}{5}\left(\frac{2e_n^2}{4\pi\varepsilon_0 R_{pp}}\right) \cong \frac{3}{5}E_F \cong \left(\frac{6}{5\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_{pp}} \\ &\cong \left(\frac{12}{5\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_0} \cong \frac{12\sqrt{3}e^2}{4\pi\varepsilon_0 R_0} \end{aligned}\right\} \tag{14}$$

$$\left(a_{asy}, a_v\right) \cong \left(\frac{10 \pm 2}{5\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_0} \cong \frac{(10 \pm 2)\sqrt{3}e^2}{4\pi\varepsilon_0 R_0}$$

c) Surface energy coefficient can be expressed as,

$$a_s \cong \frac{2a_v a_{asy}}{(a_v + a_{asy})} \cong \left(\frac{9.6}{5\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_0} \cong \frac{9.6\sqrt{3}e^2}{4\pi\varepsilon_0 R_0} \tag{15}$$

d) Congruent and Pairing energy coefficients can be expressed as

$$\begin{aligned} a_{cg} \cong a_p \cong \frac{E_F}{4} \cong \left(\frac{e_n^2}{8\pi\varepsilon_0 R_{pp}}\right) \cong \left(\frac{1}{\alpha_s}\right)\frac{e^2}{8\pi\varepsilon_0 R_{pp}} \\ \cong \left(\frac{2}{\alpha_s}\right)\frac{e^2}{8\pi\varepsilon_0 R_0} \cong \left(\frac{1}{\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_0} \end{aligned} \tag{16}$$

See Table 3 for a rough estimation of our 6 term binding energy coefficients.

Table 3. Approximate energy coefficients of the revised 6 term semi empirical mass formula

a_v MeV	a_s MeV	a_c MeV	a_{asy} MeV	a_p MeV	a_{cg} MeV
15.94	19.3	0.70	24.4	10.1	10.1
16.0	19.4	0.71	24.5	10.0	10.0

Proceeding further, binding energy point of view, considering a radius value close to the the root mean square radius of proton or neutron [44,45], for light, medium and heavy atomic nuclides, above nuclear radii can be can be approximated with the following relation. It needs further study.

$$R_{(Z,N)} \cong \left[1-\left(\frac{N-Z}{A}\right)^2\right]^x A^{1/3} \times 1.25\,\text{fm} \approx \left[\sqrt[3]{Z} + \sqrt[3]{Z^2 N}\right] \times 0.79\,\text{fm} \tag{17}$$

Clearly speaking, Coulomb energy term can be re-written as,

$$E_{cou} \cong -\frac{3Z^2 e^2}{20\pi\varepsilon_0 R_{(Z,N)}} \tag{18}$$

Binding energy expression can be re-written as,

$$BE \cong 16A - 19.4\beta A^{2/3} - \frac{1.09365 \times Z^2}{\left(\sqrt[3]{Z} + \sqrt[9]{Z^2 N}\right)} - \left[\left(1 - \frac{1}{A}\right) \times 24.5 \frac{(A-2Z)^2}{A}\right] \pm \frac{10}{\sqrt{A}} + 10 \times \exp\left(-4.2 \frac{|N-Z|}{A}\right)$$

$$\text{where } \beta \cong 1 - \left(\frac{N-Z}{A}\right)^2 \text{ and}$$

$$\frac{3Z^2 e^2}{20\pi\varepsilon_0 R_{(Z,N)}} \cong \frac{3Z^2 e^2}{20\pi\varepsilon_0} \left[\left(\sqrt[3]{Z} + \sqrt[9]{Z^2 N}\right) \times 0.79 \text{ fm}\right]^{-1} \quad \text{(19A)}$$

$$\left.\begin{aligned} BE &\cong 16A - 19.4\beta A^{2/3} - \frac{1.1 \times Z^2}{\left(\sqrt[3]{Z} + \sqrt[9]{Z^2 N}\right)} \\ &- \left[\left(1 - \frac{1}{A}\right) \times 24.5 \frac{(A-2Z)^2}{A}\right] \pm \frac{10}{\sqrt{A}} + 10 \times \exp\left(-4.2 \frac{|N-Z|}{A}\right) \\ &\text{where } \beta \cong 1 - \left(\frac{N-Z}{A}\right)^2 \end{aligned}\right\} \quad \text{(19B)}$$

Following relations (6) and (19), it can be understood that, radii of higher isotopes of higher proton numbers, seem to have a lower radii than the expected radii. See the following binding energy curves prepared for Z=28,50,82 and 114. Based on relation (6) and (19), red curve is our fit and green curve is the reference binding energy curve [19]. Based on the energy coefficients proposed in relation (7), and if $R_0 \cong 1.25$ fm, strong coupling constant [41] can be estimated as,

$$\alpha_s \cong \frac{8e^2}{20\pi\varepsilon_0 R_0 a_v} \cong 0.1151973 \text{ where } a_v \cong 16.0 \text{ MeV}$$

$$\alpha_s \cong \frac{12e^2}{20\pi\varepsilon_0 R_0 a_{asy}} \cong 0.1128463 \text{ where } a_{asy} \cong 24.5 \text{ MeV}$$

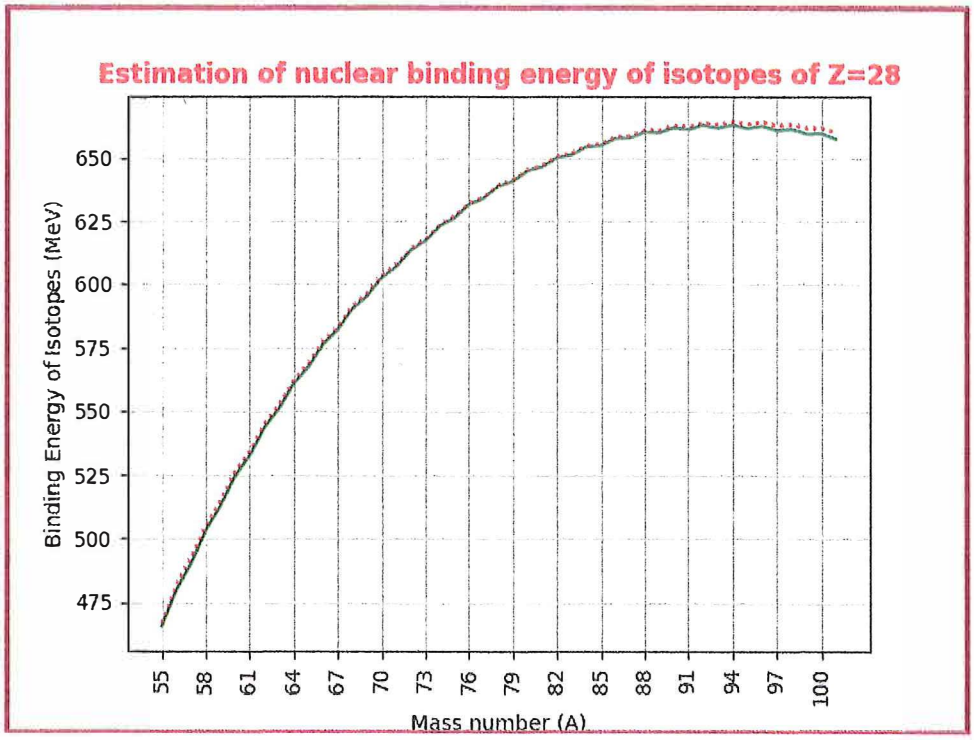
$$\alpha_s \cong \frac{9.6e^2}{20\pi\varepsilon_0 R_0 a_s} \cong 0.11401 \text{ where } a_s \cong 19.4 \text{ MeV}$$

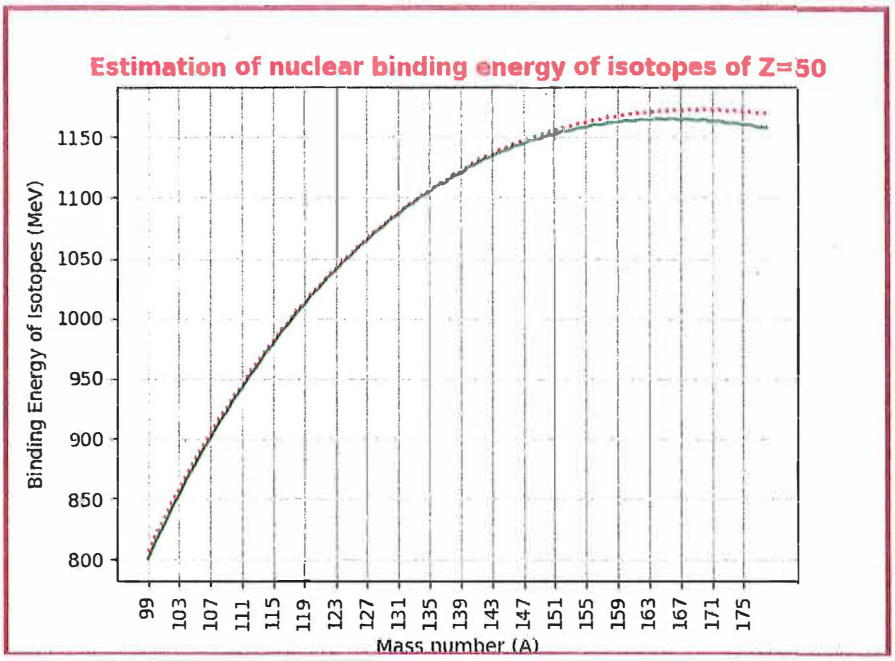
$$\alpha_s \cong \frac{e^2}{4\pi\varepsilon_0 R_0 a_{cg}} \cong \frac{e^2}{4\pi\varepsilon_0 R_0 a_p} \cong 0.1151973 \text{ where } a_{cg} \cong a_p \cong 10.0 \text{ MeV}$$

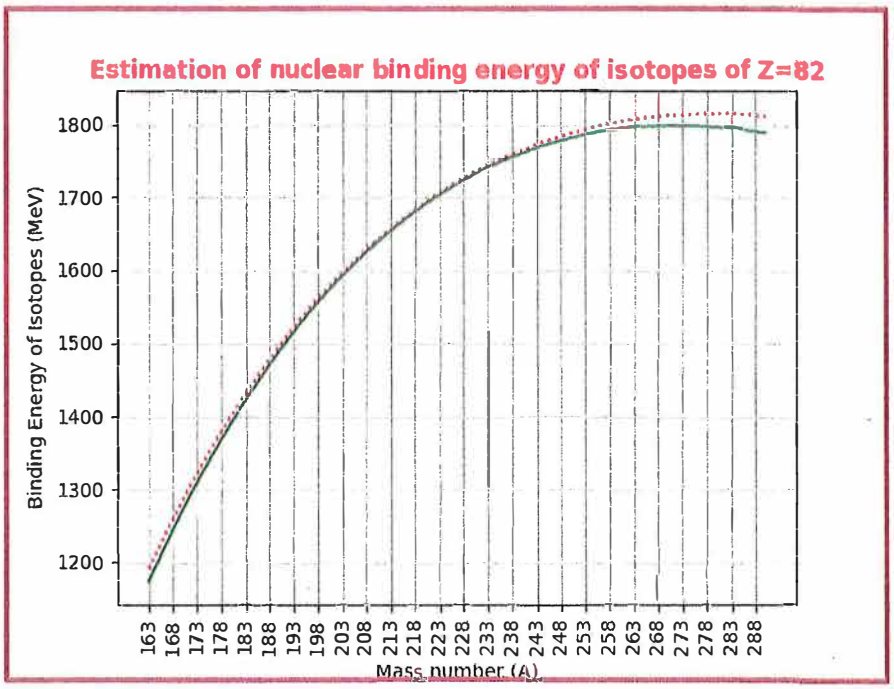
$$\alpha_s \cong \frac{1}{5\sqrt{3}} \cong 0.11547$$

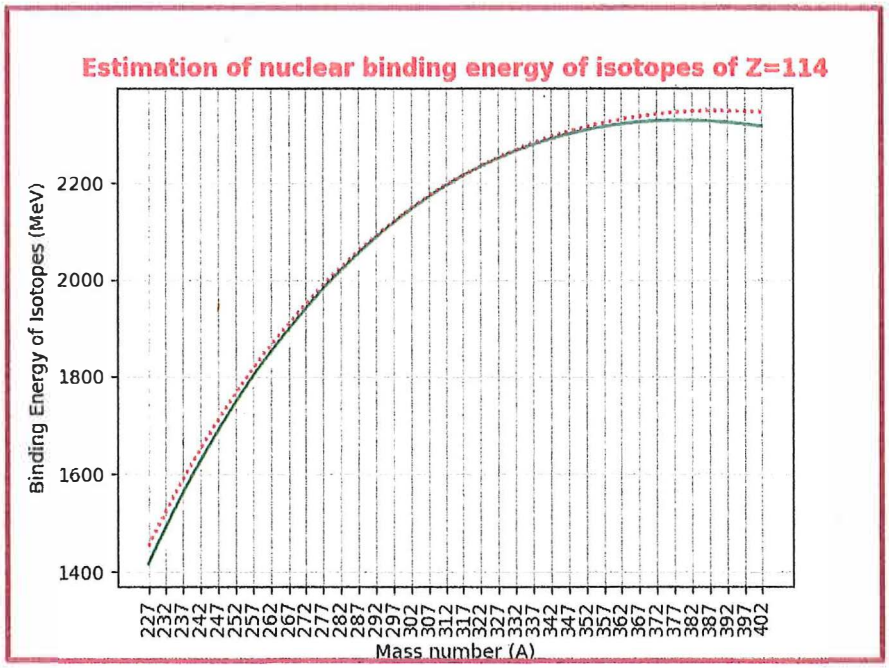
In a simplified and unified approach, approximately, it is possible to infer that,

$$\left.\begin{aligned} &(a_v, a_{asy}) \approx \left(\frac{2 \mp 0.4}{\alpha_s}\right)\left(\frac{e^2}{4\pi\varepsilon_0 R_0}\right) \\ &\frac{2a_v a_{asy}}{(a_v + a_{asy})} \geq a_s \leq \sqrt{a_v a_{asy}} \\ &a_{cg} \approx a_p \approx \left(\frac{1}{\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_0} \\ &a_c \approx \left(\frac{3}{5}\right)\frac{e^2}{4\pi\varepsilon_0 R_0} \end{aligned}\right\} \quad (20)$$




Estimation of nuclear binding energy of isotopes of Z=50
Binding Energy of Isotopes (MeV)
1150
1100
1050
1000
950
900
850
800
99
103
107
111
115
119
123
127
131
135
139
143
147
151
155
159
163
167
171
175
Mass number (A)


Estimation of nuclear binding energy of isotopes of Z=82
Binding Energy of Isotopes (MeV)
1800
1700
1600
1500
1400
1300
1200
163
168
173
178
183
188
193
198
203
208
213
218
223
228
233
238
243
248
253
258
263
268
273
278
283
288
Mass number (A)



5. Short discussion on R. M. Santilli's views on quantum mechanics and nuclear stability

By following the 1935 historical argument by A. Einstein, B. Podolsky and N. Rosen that quantum mechanics is not a complete theory [27-30], R. M. Santilli [32-34] identified a number of apparent insufficiencies of quantum mechanics in nuclear physics with particular reference to the lack of numerically exact representation (in one century of nuclear data), the prohibition by Heisenberg's uncertainty principle to represent the neutron synthesis from the electron and the proton in the core of stars despite their extremely big Coulomb attraction and the ensuing inability to represent the nuclear stability.

By following the below mentioned 4 points, he presented exact and time invariant representations of stable nuclei via the Lie-isotopic branch of hadronic mechanics and showed the necessity for the results of lifting Heisenberg's uncertainty principle for point-like particles into the isouncertainty principle of hadronic mechanics, also called Einstein's isodeterminism, for extended nucleons in condition of mutual penetration.

1) Considering protons and neutrons as isoparticles characterized by the isosymmetries and isorelativities of hadronic mechanics.
2) Representing all characteristics of the neutron at the nonrelativistic and relativistic levels as a hadronic bound state of an isoelectron and an isoproton.
3) Identifying the density of neutron in a way compatible with other experimental data.
4) Representing nuclear stability despite the natural instability of the neutron and despite the extremely repulsive protonic Coulomb forces.

It may be noted that, our approach of understanding quantum mechanics is based on 'weak gravity' and R. M. Santilli's approach is based on hadron mechanics. We are working on combining the two approaches for a better and simplified solutions.

6. Conclusion

Understanding nuclear binding energy with various physical terms and various coefficients is not a new point. With a single set of energy coefficients, our proposal helps in estimating the binding energy of all atomic nuclides starting from Z=1 to 137. Here, we would like to emphasize the point that, 4G model of the strong coupling constant (α_s) plays a vital role in understanding the origin of the binding energy coefficients and further study certainly helps in connecting high and low energy branches of nuclear physics and particle physics. Interesting point to be noted is that, all the binding energy coefficients, can be expressed as, $k\left[(2.95e)^2/4\pi\varepsilon_0(1.25\,\text{fm})\right] \cong (k/\alpha_s)\left[e^2/4\pi\varepsilon_0(1.25\,\text{fm})\right]$ where $k=(1.0\,\text{to}\,2.5)$.

Data availability statement: The data that support the findings of this study are openly available.

Conflict of interest: Authors declare no conflict of interest in this paper or subject.

Acknowledgements

Author Seshavatharam is indebted to professors Padma Shri M. Nagaphani Sarma, Chairman, Shri K.V. Krishna Murthy, founder Chairman, Institute of Scientific Research in Vedas (I-SERVE), Hyderabad, India and Shri K.V.R.S. Murthy, former scientist IICT (CSIR), Govt. of India, Director,

Research and Development, I-SERVE, for their valuable guidance and great support in developing this subject.

References

[1] Seshavatharam U.V.S and Lakshminarayana S. Radius, surface area and volume dependent electroweak term and isospin dependent asymmetry term of the strong and electroweak mass formula. Int. J. Phys. Appl. 2025;7(1):122-134.

[2] Seshavatharam U.V.S and Lakshminarayana S. Understanding the Origins of Quark Charges, Quantum of Magnetic Flux, Planck's Radiation Constant and Celestial Magnetic Moments with the 4G Model of Nuclear Charge. Current Physics. 1, e090524229812, 122-147, 2024.

[3] Seshavatharam U.V.S and Lakshminarayana S. Exploring condensed matter physics with refined electroweak term of the strong and electroweak mass formula. World Scientific News.193(2) 105-13, 2024.

[4] Seshavatharam U. V. S, Gunavardhana Naidu T and Lakshminarayana S. Nuclear evidences for confirming the physical existence of 585 GeV weak fermion and galactic observations of TeV radiation. International Journal of Advanced Astronomy. 13(1):1-17, 2025.

[5] Seshavatharam U. V. S., Gunavardhana Naidu T and Lakshminarayana S. 2022. To confirm the existence of heavy weak fermion of rest energy 585 GeV. AIP Conf. Proc. 2451 p 020003

[6] Seshavatharam U V S and Lakshminarayana S. 4G model of final unification – A brief report Journal of Physics: Conference Series 2197 p 012029, 2022.

[7] Seshavatharam U. V. S and Lakshminarayana S., H. K. Cherop and K. M. Khanna, Three Unified Nuclear Binding Energy Formulae. World Scientific News, 163, 30-77, 2022.

[8] Seshavatharam U.V.S and Lakshminarayana, S., On the Combined Role of Strong and Electroweak Interactions in Understanding Nuclear Binding Energy Scheme. Mapana Journal of Sciences, 20(1), 1-18, 2021.

[9] Seshavatharam U.V.S and Lakshminarayana S., Strong and Weak Interactions in Ghahramany's Integrated Nuclear Binding Energy Formula. World Scientific News, 161, 111-129, 2021.

[10] Seshavatharam U.V.S and Lakshminarayana S. Is reduced Planck's constant - an outcome of electroweak gravity? Mapana Journal of Sciences. 19(1), 1,-13, 2020.

[11] Seshavatharam U.V.S and Lakshminarayana S. A very brief review on strong and electroweak mass formula pertaining to 4G model of final unification. Proceedings of the DAE Symp. on Nucl. Phys. 67,1173, 2023.

[12] Seshavatharam U.V.S and Lakshminarayana S. Computing unified atomic mass unit and Avogadro number with various nuclear binding energy formulae coded in Python. Int. J. Chem. Stud. 2025;13(1):24-30.

[13] Bethe H. A. Thomas-Fermi Theory of Nuclei. Phys. Rev., 167(4), 879-907, 1968.

[14] Myers W. D. and Swiatecki W. J. Nuclear Properties According to the Thomas-Fermi Model.LBL-36557 Rev. UC-413, 1995.

[15] Myers W. D. and Swiatecki W. J. Table of nuclear masses according to the 1994 Thomas-Fermi model. United States: N. p., 1994. Web.

[16] P.R. Chowdhury, C. Samanta, D.N. Basu, Modified Bethe–Weizsacker mass formula with isotonic shift and new driplines. Mod. Phys. Lett. A 20, 1605–1618, 2005.

[17] G. Royer, On the coefficients of the liquid drop model mass formulae and nuclear radii. Nuclear Physics A, 807, 3–4, 105-118, 2008.

[18] Djelloul Benzaid, Salaheddine Bentridi, Abdelkader Kerraci, Naima Amrani. Bethe–Weizsa¨cker semiempirical mass formula coefficients 2019 update based on AME2016.NUCL. SCI. TECH. 31:9, 2020.

[19] Gao, Z.P., Wang, YJ., Lü, HL. et al., Machine learning the nuclear mass. NUCL. SCI. TECH. 32, 109, 2021.

[20] Peng Guo, et. al. (DRHBc Mass Table Collaboration), Nuclear mass table in deformed relativistic Hartree-Bogoliubov theory in continuum, II: Even-Z nuclei. Atomic Data and Nuclear Data Tables 158 (2024) 101661

[21] Cht. Mavrodiev S, Deliyergiyev M.A. Modification of the nuclear landscape in the inverse problem framework using the generalized Bethe-Weizsäcker mass formula. Int. J. Mod. Phys. E 27: 1850015, 2018.

[22] Ghahramany N, Gharaati, S., & Ghanaatian, M. New approach to nuclear binding energy in integrated nuclear model. Journal of Theoretical and Applied Physics, 6(1), 3, 2012.

[23] Ghahramany N, Sh Gharaati, Ghanaatian M, Hora H. New scheme of nuclide and nuclear binding energy from quark-like model. Iranian Journal of Science & Technology A3, 201-208, 2011.

[24] N. Ghahramany M. Ghanaatian and M. Hooshmand. Quark-Gluon Plasma Model and Origin of Magic Numbers. Iranian Physical Journal, 1-2, 35-38, 2007.

[25] C. Grojean. Higgs Physics. Proceedings of the 2015 CERN–Latin-American School of High-Energy Physics, 143-157, 2016, CERN-2016-005 (CERN, Geneva, 2016)

[26] Ahmed Abokhalil. The Higgs Mechanism and Higgs Boson: Unveiling the Symmetry of the Universe. arXiv:2306.01019v2 [hep-ph]

[27] A. Einstein, B. Podolsky, and N. Rosen. Can quantum-mechanical description of physical reality be considered complete?. Phys. Rev. 47, 777, 1935.

[28] N. Bohr. Can quantum mechanical description of physical reality be considered complete? Phys. Rev. 480, 696, 1935,

[29] J.S. Bell. On the Einstein Podolsky Rosen paradox. Physics 1, 195, 1964.

[30] J. Bell, On the problem of hidden variables in quantum mechanics. Reviews of Modern Physics. 38, 3, 447, 1966.

[31] Seshavatharam U.V.S, Lakshminarayana S. EPR argument and mystery of the reduced Planck's constant. Algebras, Groups, and Geometries. 36(4), 801-822, 2020.

[32] R. M. Santilli. Lie-isotopic representation of stable nuclei I: Apparent insufficiencies of quantum mechanics in nuclear physics
Ratio Mathematica. 52, 43-63, 2024.

[33] M. Santilli. Lie-isotopic representation of stable nuclei II: Exact and time invariant representation of the Deuteron data.Ratio Mathematica, 52, 64-130, 2024.

[34] R. M. Santilli. Lie-isotopic representation of stable nuclei III: Exact and time invariant representation of nuclear stability, Ratio Mathematica, 52,131-175, 2024.

[35] Sunil Mukhi. String theory: a perspective over the last 25 years. Class. Quantum Grav. 28 153001, 2011.

[36] Sachdev, Subir. Strange and stringy. Scientific American. 308 (44): 44–51,2013.

[37] Blumenhagen R., Lüst D., Theisen S. Basic Concepts of String Theory. Theoretical and Mathematical Physics Springer Heidelberg, Germany, 2013.

[38] Arnab Priya Saha and Aninda Sinha Phys. Field Theory Expansions of String Theory Amplitudes.Rev. Lett. 132, 221601,2024.

[39] Seshavatharam U.V.S. and Lakshminarayana S. On the compactification and reformation of string theory with three large atomic gravitational constants. International Journal of Physical Research, 9(1), 42-48, 2021.

[40] M. Ismail1, W. M. Seif, and A. Abdurrahman. Relative stability and magic numbers of nuclei deduced from behavior of cluster emission half-lives. Phys. Rev. C 94, 024316.

[41] D d'Enterria et al. The strong coupling constant: state of the art and the decade ahead. J. Phys. G: Nucl. Part. Phys. 51 090501, 2024.

[42] Maruhn, J.A., Reinhard, PG., Suraud, E. The Fermi-Gas Model. In: Simple Models of Many-Fermion Systems. Springer, Berlin, Heidelberg.2010.

[43] Chapter 7. Bulk Nuclear Properties and Nuclear Matter. https://www.physics.umd.edu/courses/Phys741/xji/chapter7.pdf

[44] H. Gao M. Vanderhaeghen, The proton charge radius. Rev. Mod. Phys. 94, 015002, 2022.

[45] G.A. Miller. Defining the proton radius: a unified treatment. Phys. Rev. C 99, 035202, 2019.

HADRONIC JOURNAL 48, 231 - 246 (2025)
DOI 10.29083/HJ.48.02.2025/SC231

THE ROLE OF CONSCIOUSNESS FOR GRAND UNIFIED THEORIES

Bertrand Wong
Dept. of Science & Technology
Eurotech, Singapore Branch
bwong8@singnet.com.sg

Received May 30, 2025
Revised June 12, 2025

Abstract

A theory of everything (TOE), or, grand unified theory (which Einstein had been working on without success, with Superstring Theory now being a good candidate), is one which unites all the forces of nature, namely, gravity, electromagnetism, the strong nuclear force and the weak nuclear force. Important as this theory might be, it is lacking in one important fundamental aspect, namely, the role of consciousness, which could in fact be considered the most fundamental aspect of physics, for example its apparently significant role and impact in quantum theory which would be explained in this paper. This paper explains that a theory of consciousness is more important than a theory of everything or grand unified theory and could potentially be the theory of everything, or, at least, a part of the theory of everything, consciousness having been the model for artificial intelligence (AI) that has become very powerful and a threat to society itself. This paper is the sequel to the paper *The Ultimate Law of Nature* which was published in Physics Essays in 2009.

Keywords: forces of nature; quantum particles; gravity; superstrings; membranes; information-processing; consciousness; unification

PACS: 04.20.Cv, 11.30.Pb & 89.20.Ff

Copyright © 2025 by Hadronic Press, Inc., Palm Harbor, FL 34684, USA

1 Overview

Einstein had tried very hard to unify the four forces of nature, namely, gravity, weak nuclear force, strong nuclear force and electromagnetism to arrive at a unified field theory but had failed. He had purportedly tried to make use of a set of 16 complex tensor equations, the combinations of ten of which representing gravitation and the remaining six representing electromagnetism, with the idea that a pure gravitational field could exist without an electromagnetic field but a pure electromagnetic field could not exist without an accompanying gravitational field. His field equations are as follows:-

$$G_{uv} + g_{uv} A = \frac{8\pi G}{C^4} T_{uv} \qquad (1)$$

where:

G_{uv} represents the curvature of space-time

g_{uv} represents the structure of space-time

A (lambda) is the cosmological constant, a term which could describe a repulsive force throughout space – this term, represented by the Greek letter lambda, had been included by Einstein in his general relativity equations which describe how matter and energy bend space-time; A (lambda) might be the result of vacuum energy, the energy in empty space, made of "virtual particles" – pairs of particles and anti-particles which constantly appear and disappear, wherein the particles and anti-particles inter-act and annihilate each other; in turn A (lambda) might cause dark energy which is the force responsible for the acceleration of the expansion of the universe resulting in galaxies flying apart

G is the gravitational constant

T_{uv} represents the energy and momentum of matter and radiation

C is the speed of light

He was not able to derive the electromagnetic field equations, even for the weak-field approximation, and had no success at all with the unified field theory, which would be the theory of everything. [1-4]

This paper describes the behavior of quantum particles and provides the possible explanations for it for example their relationship with consciousness, and also advocates the modification and improvement of the theory of everything so that it would be more encompassing and serve a wider, more important purpose such as providing a deeper understanding of consciousness which is the essence of human nature so that humans benefit from this understanding and would be better for it such as having improvement of intellectual capacity and creativity, for example being more analytical and being better at processing information (as a matter of fact, we humans have created and developed artificial intelligence (AI) modeled after consciousness, which has become so powerful that it has become a threat to society, for instance by displacing humans from their jobs by taking over their jobs, being able to perform the jobs much faster and better such as solving complex mathematical problems, beating chess grandmasters and champions in chess, composing songs, painting pictures, writing books, etc.), having better understanding of human nature in order to reduce or minimize conflicts, possibly even controlling the working of consciousness and make it work better so that it would benefit society, etc., besides providing a deep understanding of the external environment, for example, the nature of particles, wherein this understanding enables humans to utilize these wonders of nature for their own benefit, for example, generation of nuclear energy, having faster computers through quantum computing utilizing quantum entanglement, etc. Consciousness represents our internal nature while matter and particles represent nature external to us. Leaving out consciousness from nature thus gives an incomplete picture of the physical world, that is, the internal part of

nature is missing while the external part is there. Also, consciousness is very important and is the essence of life or existence – consciousness is life itself (scientists have reportedly even found evidence of consciousness in plants, such as plants responding to certain stimuli). We may be able to live without quantum particles, for example, but we cannot live without consciousness. As there is also an apparent link between consciousness and particles (consciousness and particles seem to have a symbiotic relationship), the paper would also analyze this; this apparent link between consciousness and quantum particles implies that a theory of everything should include a theory of consciousness to be complete. The paper would thus make a strong case for consciousness though matter and quantum particles are not to be diminished in importance. It is therefore not surprising that physicists are now studying consciousness to unravel the mysteries of the universe. [5-6]

2 On Quantum Particles and Their Behavior

It is apparent that consciousness is somehow connected with phenomena in the quantum realm. Consider Bell's double-slit experiment. In this experiment, are the particles and waves just sensitive to screens and other equipment or are they picking up messages from the physicist's brain? It could be said that particles of matter and particles of mind might come into being together, but in any case our self-awareness and what seems to be some kind of consciousness at the quantum level appear to be in deep communication. David Bohm had in his classic work, *Wholeness and the Implicate Order*, developed a theory of quantum physics which treats the totality of existence, including matter and consciousness, as an unbroken whole. [7] There is the implication that at the sub-quantum level the observing device used to measure the quantum particles must have connections with all parts of the system, including the link with our consciousness, and through these a "signal" might be transmitted to the molecule that a certain observable was eventually going to be measured. [8-14] Consciousness is non-local; in other words, no one could say where the mind is or how far the effects of thought could reach. As a matter of fact, within the

brain, all the forces are active, including gravity [15], which is the force that holds the entire universe together – however gravity appears to have little effect on quantum particles.

Though fundamentally the material brain and the other matters are comprised of the same thing, namely, atoms, reductionists might wonder why the material brain as compared to other matters is so special. Is this due to the special composition of the atoms in the brain? If we take a number of atoms and arrange their composition so that they would be similar to that of the material brain, could we produce a brain, and, consciousness?

The universe might be in some sense a Great Mind and a theory of everything might have to include a theory of consciousness. Superstrings, which are a strong candidate for a theory of everything, might be thought particles with a life of their own. Many physicists are making attempts at deriving a Grand Unified Theory of the universe on the basis of particle physics. This effort might be incomplete as particles might be just a reflection of the information-processing foundations of the universe (but it is certainly not a waste of time as this research might help us to figure out how the information-processing system works). In the last analysis, we might not be able to completely understand the universe, if it is ever possible to do so, until it is examined as a self-evolving and organizing information-processing machine, one which produces intelligent minds to examine itself with. Hence, a theory of consciousness might be consolidated with the theory of physics (such as the Superstring Theory or the Membrane Theory) into a Grand Information Theory (GIT). This could be considered the Theory of Everything. [16]

We examine several important ideas, including some which had helped their originators to win Nobel prizes. Nobelist Schrodinger had found an equation which could be applied to any physical system in which the mathematical form of the energy is known, which is as follows:-

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{8\pi^2 m}{h^2} (E - V) \psi = 0 \qquad (2)$$

where ∂^2 is the second derivative with respect to x, x is the position of the particle, ψ is the Schrodinger wave function, or, the probability amplitude for an electron in the state n to scatter into the direction m, E is energy and V is potential energy.

The Schrodinger equation is a deterministic time-symmetrical description of nature. In classical mechanics, when we say that a quantum system is in a particular "state", we mean that the state is a point in phase space. It is here described by a wave function whose evolution over time is expressed

by the following equation:-

$$ih / 2\pi \, \partial\psi (t) / \partial t = H_{op} \, \psi (t) \qquad (3)$$

This equation identifies the time derivative of the Schrodinger wave function ψ with the action of the Hamiltonian operator on ψ. It is not derived but assumed at the start, and could thus be validated only by experiment. In quantum theory, it is the fundamental law of nature. Here, ψ is the probability amplitude for an electron – it is only an abstraction (a function of consciousness, having no physical reality). ψ is also, in a sense, the electron's own intensity wave. When it is squared and the absolute value is taken, it turns out to be a physical probability of the associated particle's presence. Later, Born stated that the probability of the existence of a state is given by the square of the normalized amplitude of the individual wave function (i.e. ψ^2). This was another new concept, that is, the probability that a certain quantum state exists. Born had said there were no more exact answers in atomic theory, but just probabilities. The wave Ψ determines the likelihood that the electron would be in a particular position, and, unlike the electromagnetic field, has no physical reality. The formal solution of the Schrodinger equation is:-

$$\psi (t) = U (t) \, \psi (0) \qquad (4)$$

where $U(t) = e^{-iHt}$, $U(t)$ is the evolution operator that links the value of the wave function at time t to that at the initial time $t = 0$. Both future and past play the same role, since $U(t_1)\,U(t_2) = U(t_1 + t_2)$, whatever the sign of t_1 and t_2. This property defines a dynamical group.

In quantum mechanics, the behavior of particles, which are regarded as waves, could be predicted, as it were, and, they are thus known as probability waves or Dirac wave particles. Here, there is a wave/particle duality. When the particle is not observed (when consciousness is not present), it remains a wave (a probability wave), but on being observed (when consciousness is present) it becomes a particle.

We here consider the famous Schrodinger's Cat experiment. In the experiment, a cat placed in an enclosed space with a poison vial might be either dead or alive when the cover or lid is opened – the cat's life depended on whether the poison vial was broken by a hammer or not. When the cover is opened, either of the two following outcomes would be discovered:-

[1] The poison vial had been broken by the hammer and the cat had been killed off by the poison.
[2] The poison vial had remained intact and the cat had lived.

There is a 50/50 chance that the cat either lives or dies when the cover is opened. Our mind or consciousness is aware of this probability when it is aware of the contents of the container. If, on the other hand, the cover is never opened and our consciousness is never aware of the cat and the poison in the container, the probability of the cat dying of poisoning or carrying on living would never have occurred to our consciousness. Hence, the importance of the role of consciousness in this experiment as well as in quantum mechanics (where the scenario is similar). [17]

According to the Uncertainty Principle, for which Werner Heisenberg won a Nobel prize, the very act of observing a quantum particle affects its behavior, that is, consciousness affects a quantum particle. According to this theory, the position and the momentum of an elementary particle could not be known

simultaneously. The reason for this is that if an electron could be held still long enough for its position to be determined, then its momentum could no longer be determined. A special point is that the product of two uncertainties (or spreads of possible values) is always at least a certain minimum number. From the de Broglie/Einstein relation, $\Delta p \sim h/\lambda$, Heisenberg obtained the imprecision in the momentum. Multiplying the two inaccuracies together, he showed that the product, $\Delta x\, \Delta p$, would always be greater than or equal to ($\geq$) a certain amount, as follows:-

$$(\Delta x)\,(\Delta p) \geq (\lambda)\,(h/\lambda) \geq h, \text{ or, } \ldots\ldots \qquad (5)$$

$$\Delta x\, \Delta p \geq h \qquad (6)$$

where Δp and h/λ represent the de Broglie relation, and, Δx and λ are from the diffraction limit.

The frustrated researcher seeking certainty must always make a compromise, knowledge gained about time, for instance, is paid for in uncertainty about frequency and vice versa. Though we do not notice Heisenberg Uncertainty Principle in our everyday experience with the gross macroscopic world, the wave/particle duality defeats the atomic experimentalist who seeks perfection.[18]

Pauli, who was a Nobelist, was fascinated by subatomic particles and consciousness, collaborating for some time with psychologist Carl Jung, whose patient he was for a time. [19] The mathematician, John von Neumann, the biologist, George Wald, and the physicists, David Bohm and Arthur Eddington, had declared that the universe is mind stuff. The mathematical physicist, Sir Roger Penrose, Nobel laureate and sometime colleague of Stephen Hawking, considers that there is "definite possibility" that consciousness is connected with phenomena in the quantum realm. The Anthropic Principles (both strong and weak) stipulate that in man there are intellectual capacities which are there for a reason, that somehow human beings with their minds are obliged to help the universe through the next stage; this is indeed manifested by the fact that scientists such as Einstein, etc., had been using their brains/consciousness to

understand nature and many had been attempting to formulate a theory of everything or unified field theory. [20-21] The Gaia Hypothesis of biologist, James Lovelock, paints a picture of the earth functioning as one large organism, which implies will and consciousness being at work. [22]

A number of scientists had postulated that there has to be a "cosmic consciousness" pervading the universe; objects spring into existence when measurements are made, measurements which are made by conscious beings, which implies that there must be cosmic consciousness that pervades the universe determining which state we are in. Some scientists, for example, Nobel laureate Eugene Wigner, had argued that this is evidence of the existence of God or some cosmic consciousness. Wigner had remarked that it was not possible to formulate the laws of quantum theory in a fully consistent way without reference to consciousness.

Classical philosophers such as Berkeley and Hume had in fact questioned whether the existence of any object was independent of the existence of the mind or consciousness: If I had never seen (never been aware of) an object, does that object exist?

Thus the great importance of the role of consciousness in nature. In fact, there appears to be an intricate link between nature and consciousness, the latter being apparently the common denominator in the workings of nature, especially at the quantum level, as is described above. However, far-fetched as it might seem, consciousness could be some sort of particles, not unlike quantum particles, which could explain why they are able to interact with one another, as is afore-described. This could also possibly explain phenomena such as intuition, mind-reading and telepathy. For example, Rupert Sheldrake, a well-known biologist, in his "morphogenesis" theory, stated that all our minds or consciousness are linked or interconnected, so that how people think and behave in one geographical area affects how people living in another distant geographical area act and think without any communication whatsoever between them, a phenomenon which applies to animals as well. Consciousness could therefore be regarded as a "force" of nature. [23]

We question here: What is life, the theory of everything or the grand unified theory without consciousness? This is in fact an irrelevant question as it is a

tautology, for life and consciousness are synonymous, and, without life or consciousness to contemplate a theory of everything or grand unified theory, the latter is an impossibility and is redundant.

3 Conclusions

The discussion above more or less shows incontrovertibly that consciousness plays a highly important role in the physical world. It is hence of great importance to have a theory of consciousness, which should precede a theory of everything or grand unified theory, for to contemplate a theory of everything or grand unified theory without taking into account the evident role of consciousness is like riding the horse-cart without the horse. It should be more important to understand ourselves, our consciousness, our nature better, an understanding we are still evidently lacking, rather than spend an inordinate amount of time trying to understand better the nature external to us, that is, consciousness should be more important to us than external nature as consciousness is in reality life or existence itself. It is evident that it is mind which controls external matter and not vice versa, for otherwise life would be overwhelmed by natural phenomena. Therefore, a theory of consciousness could potentially be the theory of everything (consciousness evidently being capable of controlling everything in nature including itself, exhibiting great intelligence), or, at least, a part of the theory of everything, a highly important, fundamental part, for example, as a Grand Information Theory (GIT), which is described above, consciousness and matter having some fundamental link as is also described above. Importantly, this theory of consciousness giving us a more comprehensive understanding of our mind would be a boon or aid to our affairs. [24]

We provide some possible explanations for the above-described effect of consciousness on quantum particles as well as for the behavior of quantum particles, which are as follows:-

[1] As is explained earlier, the brain itself, which produces consciousness (as well as the person's body, etc.), is also matter, and it (and the person's

body, etc.) and the other matters in the universe are comprised of the same thing, namely, atoms, or, particles. It is possible that in the presence of consciousness, that is, the material brain itself (and/or the person's body, etc.), the particles in the brain (and/or the body, etc.) interact or engage with the other particles outside it affecting one another.

[2] (a) Like a broadcasting station which transmits radio and television signals which are received by the antennas of radios and televisions over very long distances simultaneously and instantaneously, quantum particles (which may be particles of matter, brain/consciousness, body, etc.) might be "broadcasting stations/transmitters" cum "antennas/receivers" capable of transmitting and receiving signals instantaneously between themselves, that is, quantum entanglement which is still a great mystery occurs. We elaborate further on how entanglement between two quantum particles might work. Even if a pair of particles were separated by a huge distance, the measuring of one particle's spin would result in the other particle's spin automatically resolving itself in the other direction, this effect occurring instantaneously, apparently breaching the velocity of light and the rules of relativity (wherein the velocity of light is the ultimate velocity for any quantum particle which is described by the following equation where c represents the velocity of light:

$$v = (c + c) \div (1 + c.c/c^2) = 2c/2 = c! \text{ (And not } 2c!) \quad (7)).$$

The two entangled particles might be linked by some kind of electromagnetic "force/link", the analog of which is the mechanical system of two similar physical objects linked by a rod. For instance, one of these two similar objects is directly joint or connected to one end of the rod while the other object is joint to the other end of the rod through two similar interlocking gears which are mechanically arranged in such a way that the turning of one of these objects at one end of the rod by a certain fraction of a revolution in one direction would result in the object at the other end of the rod turning by the same fraction of a revolution in the opposite direction instantaneously. What happens is that turning, for example, the first object joint or connected directly to one end of the rod would turn the rod in the same direction by the same fraction of

a revolution at the same instant, the rod would turn the first gear joint to it at its other end in the same direction by the same fraction of a revolution at the same instant, this gear would turn the similar gear interlocked with it in the opposite direction by the same fraction of a revolution at the same instant, and, as the other object is joint to this second gear that turns in the opposite direction the other object itself also turns in the opposite direction by the same fraction of a revolution at the same instant (all these various actions taking place at the same time, all at once, simultaneously). A similar turning action on the second object joint to the other end of the rod would now result in the first object joint to the other end of the rod turning in the opposite direction by the same fraction of a revolution instantaneously. The following describes how the above-stated mechanical principle might apply to the behavior of the two entangled particles. Any spin motion (measured) in one of the particles may theoretically cause instantaneous motion (for example, spin or vibratory) in the electromagnetic "force/link" that links this particle to the other particle (as per the case of the first object and the rod in the above-described mechanical example). This instantaneous motion of the electromagnetic "force/link" may theoretically effect instantaneous motion in the other particle (as per the case of the rod and the second object in the above described mechanical example) which may spin in the opposite direction (as it has been conditioned to do so through the entanglement process in accordance with the Pauli exclusion principle). (Note: The motions of the two entangled particles and the electromagnetic "force/link" may theoretically take place simultaneously, at the same instant or instantaneously (as is in the case of the moving objects/parts in the above-described mechanical example).)

(b) The two entangled particles may theoretically be simultaneously controlled by a "brain" or "controller" (consciousness). This "brain/controller" may theoretically issue a signal to both particles at the same instant causing them to act as they do at the same instant. This is comparable, for example, to a computer issuing a command to two printers (or other equipment) at the same instant causing the two printers to print at the same instant (parallel processing comes to mind), with the two printers programmed to respond differently to the same command at the same instant (for example, one printer prints blue ink in

response to a command while the other printer prints red ink in response to the same command at the same instant). (This may also be likened to the case whereby a light switch controls two (or more) light bulbs such that when the light switch is turned on the two (or more) light bulbs are lighted simultaneously. Entanglement of two particles is rather similar to the programming of an equipment with a computer resulting in the two particles acting the way they are expected or "programmed" to.)

(c) Information from one of the two entangled particles may theoretically be carried to the other particle by an extremely fast carrier wave that travels faster than the velocity of light (tachyon) causing the other particle to act with an opposite spin at practically the same instant. (Note: Since the speed of this carrier wave theoretically exceeds the velocity of light and light may be required to detect it, it may be undetectable. As quantum entanglement may be the result of tachyons, that is, faster-than-light particles, at work, it may thus signify the existence of tachyons, which is another outstanding challenge in physics.)

(d) There may theoretically be an unknown influence, a mysterious undiscovered force, at work bringing about quantum entanglement.

[3] The following is another possible explanation, namely, consciousness could be some sort of particles, not unlike quantum particles, which could explain why they are able to interact with one another (that is, consciousness interacting with quantum particles), as is described above, which could also possibly explain phenomena such as intuition, mind-reading and telepathy; all our minds or consciousness are possibly linked or interconnected, and, consciousness is apparently a "force" of nature.

The above-mentioned possibilities are worth looking into.

It is thus not surprising that physicists now take consciousness more seriously and are studying consciousness in order to unravel the mysteries of the universe. The problem is of course that consciousness is intangible and abstract compared to the objects of nature, for example quantum particles which are tangible, making it harder to unravel its mystery; unless consciousness could be proved to be linked to some tangible consciousness

particles or some tangible quantum particles (wherein it is possible to describe with mathematical equations the interactions of these quantum particles of consciousness with the quantum particles of matter), quantitative or mathematical modeling of consciousness would be difficult though the manifestations of consciousness are evident all over the place; it is probably not far from the truth to say that consciousness is as mysterious as God who is commonly deemed the creator of the universe. That is, discovering the truths about consciousness which is intangible is apparently more difficult than discovering the truths about matter which comprises of tangible quantum particles, though it is evidently an important undertaking, perhaps the most important undertaking by physicists or scientists. However, we could find assurance in the fact that physicists are now studying consciousness to unravel the mysteries of the universe such as having deeper understanding of the behavior of quantum particles wherein consciousness and quantum particles are apparently closely linked, as is explained above; this apparent close link between consciousness and quantum particles implies that a theory of everything should include a theory of consciousness to be complete.

Moreover, as is explained above, consciousness is the "internal aspect" of nature while matter and particles are the "external aspect"; this means that leaving out consciousness would give an incomplete picture of nature, that is, unifying the forces of nature excluding consciousness would not give a theory of everything but a theory of partial things. As consciousness falls under the purview of biology which is the science of living things, as is evident in the above explanations, a theory of everything might have to unify physics and biology.

References

[1] P. Davies, *Superforce (The Search for a Grand Unified Theory of Nature)*, Counterpoint, 1984
[2] J. Gribbin, *The Search for Superstrings, Symmetry, and the Theory of Everything*, Back Bay Books, 2000
[3] F. D. Peat, *Superstrings and the Search for the Theory of Everything*, Contemporary Books, 1988
[4] S. Weinberg, *Dreams of a Final Theory (The Search for the Fundamental Laws of Nature)*, Vintage, 1993
[5] L. Krauss, *The Known Unknowns: The Unsolved Mysteries of the Cosmos*, Head of Zeus Ltd, 2023
[6] G. Musser, *Putting Ourselves back in the Equation: Why Physicists are Studying Human Consciousness and AI to Unravel the Mysteries of the Universe*, Farrar, Straus and Giroux, 2023
[7] D. Bohm, *Wholeness and the Implicate Order*, Routledge, 1980
[8] A. Beiser, *Concepts of Modern Physics*, Fifth Edition, McGraw-Hill, 1995
[9] M. Chown, *The Quantum Zoo: A Tourist's Guide to the Neverending Universe*, Joseph Henry Press, 2006
[10] P. A. M. Dirac, *The Principles of Quantum Mechanics*, Oxford University Press, 1958
[11] P. A. M. Dirac, The Evolution of the Physicist's Picture of Nature, *Scientific American*, May 1963
[12] E. Merzbacher, *Quantum Mechanics*, Second Edition, John Wiley & Sons, 1970
[13] A. Modinos, *Quantum Theory of Matter*, John Wiley & Sons, 1996
[14] A. Vilenkin, Predictions from Quantum Cosmology, *Physical Review Letters*, 1995, Volume 74, 846-849
[15] B. Wong, Interpretation of Gravity, *Hadronic Journal*, 2021, Volume 44, Issue 4, 329-341
[16] B. Green, *The Elegant Universe*, Vintage Books, 2000
[17] J. Gribbin, *In Search of Schrodinger's Cat (Quantum Physics and Reality)*, Bantam Book, 1984

[18] J. B. Hartle & S. W. Hawking, The Wave Function of the Universe, *Physical Review*, 1983, Volume D28, 2960-2975
[19] D. Lindorff, *Pauli and Jung: The Meeting of Two Great Minds on the Unity of Matter and Spirit*, Quest Books, 2004
[20] R. Penrose, *Shadows of the Mind: A Search for the Missing Science of Consciousness*, Oxford University Press, 1996
[21] R. Penrose, *The Emperor's New Mind: Concerning Computers, Minds, and the Laws of Physics*, Oxford University Press, 2002
[22] J. Lovelock, *Gaia: A New Look at Life on Earth*, Oxford University Press, 2000
[23] R. Sheldrake, *A New Science of Life: The Hypothesis of Morphic Resonance*, Park Street Press, 1995
[24] B. Wong, The Ultimate Law of Nature, *Physics Essays*, 2009, Volume 22, Issue 3, 296-300

ISSN: 0162-5519

Established in 1978 by Prof. R. M. Santilli at Harvard University Hadronic Journal and Algebras, Groups and Geometries have been regularly published since 1978 without publication charges and are among the few remaining independent refereed journals.

This Journal publishes advances research papers and Ph. D. theses in any field of mathematics without publication charges.

For subscription, format and any other information please visit the website http://www.hadronicpress.com

HADRONIC PRESS INC.
35246 U. S. 19 North Suite 215
Palm Harbor, FL 34684, U.S.A.
http://www.hadronicpress.com
Email: info@hadronicpress.com
Phone: +1-727-946-0427

HADRONIC JOURNAL Volume 48, Number 2, June 2025

HADRONIC PRESS, INC.